INSTITUTE LEVEL PLANNING FOR ADMISSIONS—A QUANTITATIVE APPROACH

By

LAXMI CHAND MEHTA

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DEPARTMENT OF MECHANICAL ENGINEERING
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LAXMI CHAND MEHTA

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TO MY PARENTS

CERTIFICATE

Certified that this work on "INSTITUTE LEVEL PLANNING FOR ADMISSIONS - A QUANTITATIVE APPROACH" by Laxmi Chand Mehta has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

DR. J.L. BATRA
Assistant Professor
Department of Mechanical Engineering
Indian Institute of Technology Kanpur

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SYNOPSIS

LAXMI CHAND MEHTA
M.Tech.(Mech.)
Indian Institute of Technology Kanpur
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INSTITUTE LEVEL PLANNING FOR ADMISSIONS- A QUANTITATIVE APPROACH

The work presented here pertains to an educational planning problem of admissions for postgraduate students for Master of Technology at Indian Institute of Technology, Kanpur. Two models have been developed for forecasting the number of students who are likely to join the various engineering disciplines. The proposed models are based on the Linear Cyclic Forecaster and Kalman Filter Algorithm. In the Linear Cyclic Forecaster, the number of students who are likely to join the various disciplines is considered as deterministic while in Kalman-Filter it is considered as probabilistic.

Dynamic programming has been used for determining the number of seats to be provided for the various disciplines during the forthcoming periods or semesters. The optimization was carried out using the criterion of minimizing the cost function subject to various constraints. The constraints considered are : Financial resources available and the permissible values of the minimum and maximum number of seats available in each discipline.

The proposed models have been validated by using the data furnished by the Post-graduate Office of I.I.T. Kanpur.

CHAPTER I

INTRODUCTION

1.1 GENERAL INTRODUCTION

It is widely accepted that some measure of educational planning is necessary to ensure that the educational system discharges its heavy responsibilities to the society in an efficient manner. In India, the expenditure on education forms a significant portion (about 9%) of nation's economic resources. Further it is believed that if planned properly, education will make very important contributions to the economic growth and the social development of the country. Due to the complex nature of our society, there are very many conflicting objectives which the planner has to bear in mind. These conflicting objectives may result from i) rapidly increasing demand by the society for literacy, ii) equality of opportunity, iii) manpower requirements for the growth of economy and iv) individual's freedom of choice in the type of education he/she wants to pursue. The belief that education is an important contributor to the economic growth has very recently triggered a lot of interest amongst researchers to come up with methodologies for effective educational planning.

Some of the important questions relating to the educational planning are :

i) What amount of society's resources should be devoted to education?

ii) What should be the ratio amongst the educated people of various disciplines keeping in view the requirements of the country?

iii) How should the total resources be distributed among various types and disciplines of education?

iv) What would be the likely rate of return for the expenditure incurred on an educational system?

These questions pertain to the economic efficiency of educational planning of the country as a whole. Since educational systems are very complex in nature, no attempt has been made in the past to find optimal solutions for education planning problems on national level. Keeping in view the intricacies of educational planning, many research workers have tried to solve some of these problems at institute or university level by various techniques like simulation [20], PERT/CPM [28], linear programming [2], control theory [23, 3], etc. These techniques have primarily been used to study the flow of students through an educational system without considering any resource constraints, allocation of resources to the various facilities of an educational institution, scheduling students to classes, development of curriculum, etc.

However, it seems that no-body so far has studied the apportioning of financial resources keeping in view the interaction of various disciplines among which these resources have to be distributed. Further no research has been reported for forecasting the requirements of the various disciplines taking into account the

effect of :

i) salary gradient* for the graduate of various disciplines

ii) the number of students called for interview in these disciplines

In this thesis an attempt has been made to study a subsystem of the total educational planning system. The subsystem selected is the planning of the post-graduate technical education at the Indian Institute of Technology, Kanpur taking into account the effect of above mentioned two factors. Mathematical models have been proposed and tested to tackle the following problems :

1. Forecasting for enrolment in the various disciplines of engineering.

2. Number of seats to be provided in the various disciplines taking into account various constraints.

For forecasting two models have been proposed. The first one is a linear cyclic forecaster and other is based on the principle of Kalman Filter. For predicting the number of seats in the various disciplines, dynamic programming is used to advantage.

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* This is the difference between the average salaries offered to a postgraduate and an undergraduate student.

CHAPTER II

LITERATURE REVIEW

The review of the literature on the educational planning suggests that the models can be grouped into the following four categories.

2.1 CONTROL THEORY TYPE MODEL

Koenig and Keeney [23] developed a model which can be used at the university level for the distribution of educational resources to efficiently serve a number of students. Alper and Smith [3] developed a model for finding number of places to be made available in a sector of education to satisfy an unknown social demand. Besides the control theory concepts, dynamic programming was used to determine the sequence of decisions which optimizes the cost criterion. The major assumption in their work is that the number of potential entrants is limited to an unknown constant level. One might question the feasibility of such an assumption.

2.2 SCHEDULING MODEL

Blackesley [12] developed models for scheduling students to classes at the beginning of each term, in data processing of student grades, records, etc. Abell [1] developed optimum seeking algorithms to schedule final examinations of an educational institute. Taft and Reisman [37] developed a model which determines the sequences of courses, for a student in an academic

institution which maximizes mastery levels at graduation.

2.3 NETWORK MODEL

Myrick, J.A. [28] developed a network model of the Purdue Freshman Engineering curriculum. The model has been programmed for simulation using the GASP II procedure. Curriculum, for a student can be considered as a time based precedence graph, where the arcs represent the event that a course has been completed. Figure 2-1, for example, represents one of the two semester curricula available to a student in the Purdue Department of Freshman Engineering. A network can represent an individual student or a group of students taking the same curriculum.

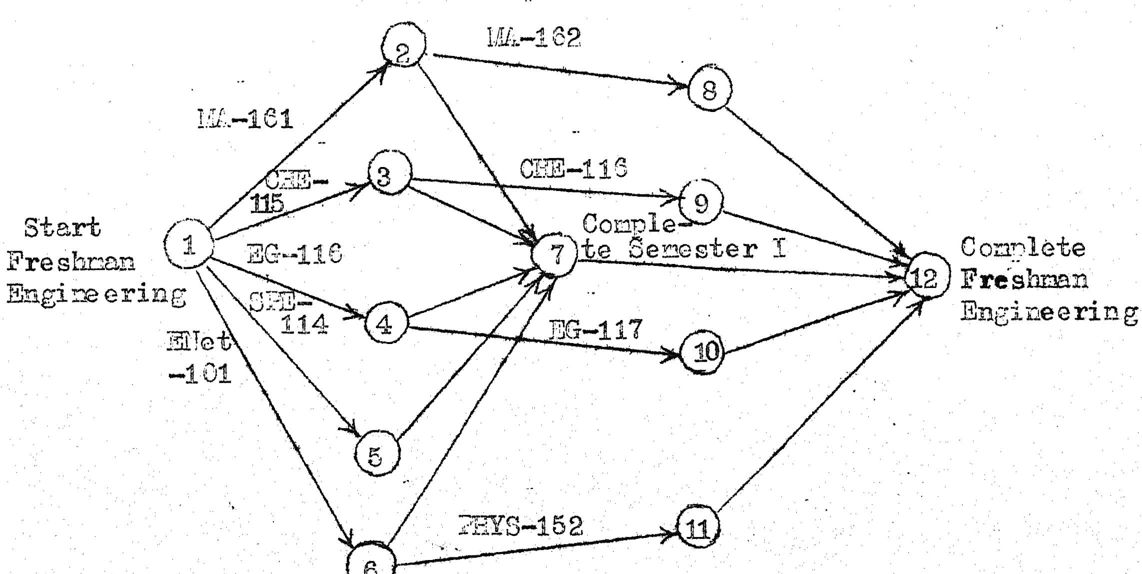

Fig. 2-1

THE NETWORK MODEL OF ENGINEERING CURRICULUM

2.4 OTHER MODELS

Judy [20] developed simulation model to solve educational problems of resource allocation. Sanderson [33] developed a model to study the impact of increased enrolments on educational facilities. In 1967, educators, statisticians, engineers and other professionals met in Washington in the first U.S.A. conference devoted exclusively to the application of operations research techniques to the field of education.

Pritsker [30] used decision theory approach for enrolment policies of Arizona State University. His objective was to predict the number of students expected to be enrolled in future. For this he minimized the total cost keeping in view a fixed financial budget. Different penalties were specified for poor predictions - both overprediction and underprediction. The two terms used above and the associated penalties in terms of cost are delineated in the subsequent sections.

1. Underprediction :

The term underprediction implies a predicted value less than the actual value. It was further divided into the following two categories :

a) Major underprediction : If the predicted value is less than a specified percent of actual value, Pritsker defines it as major

underprediction. Herein a request has to be made to grant funds over and above the scheduled amount. Thus a cost/underpredicted student was indicated by a constant C_1.

b) Minor underprediction : If the predicted value is less than a specified percent of actual value but greater than major underprediction, Pritsker defines it as minor underprediction. Herein no request has to be made to grant funds over and above the scheduled amount and a penalty due to operation on a lower budget is assessed. The cost/underpredicted student was indicated by a constant C_3.

2. <u>Overprediction</u>

The term overprediction implies a predicted value greater than the actual value. It wasfurther divided into the following two categories :

a) Major overprediction : If the predicted value is greater than a specified percent of actual value, Pritsker defines it as major overprediction. Herein a request has to be made to refund funds over and above the scheduled amount. Thus a cost/overpredicted student was indicated by a constant C_2.

b) Minor overprediction : If the predicted value is greater than a specified percent of actual value, but less than major overprediction, Pritsker defines it as minor overprediction. Herein no request has to be made to refund funds over and above the scheduled amount. Thus a cost/overpredicted student was

indicated by a constant C_4.

An analytical approach gave the best predictor of enrolment to be made in future. A sensivity analysis was carried out for various values of C_1/C_2 = 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0, 9.0.

A recently developed technique of Kalman filter based on the principle of control theory can also be used for prediction of enrolment. Kalman [21] considered the linear estimator for estimating the states of a nonlinear control system. Kalman defines noise corrupted observations in a system as a nonlinear system. Kalman considered the linear estimator as a linear function of observation data and stated that the best linear estimator is best of all estimators (linear or nonlinear). Estimation of the state in the presence of unknown parameters of the nonlinear system was treated by Magill [24] and Hilborn and Laintotis [18]. Cox [16] and Balkrishnan [6] treated same problem via a maximum likelihood criterion.

CHAPTER III

PROBLEM FORMULATION

3.1 INTRODUCTION

This study on the planning of an educational system for a technical institution involves the prediction of the number of seats to be provided for the various engineering disciplines keeping in view the socio-economic constraints. For this the objective function at any period+ k, Z_k, is expressed by the equation :

$$Z_k = f\,(U_k^j\ ,\ X_k^j\ ,\ C_1^j\ ,\ C_2^j\ ,\ F_k^j\ ,\ B_k^p\ ,\ G_k^j) \qquad (3.1)$$

$$j = 1,2,\ldots,n$$

$$k = 1,\ldots\ldots,N$$

where,

Z_k = the value of the objective function in terms of cost units for the k^{th} period,

X_k^j = the number of students who are likely to join the discipline j for the k^{th} period,

U_k^j = the number of seats to be provided in the discipline j for the k^{th} period,

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+ In this work one unit of period is taken to be a semester.

C_1^j = the penalty in terms of cost units for under-prediction* for the discipline j

C_2^j = the penalty in terms of cost units for over-prediction** for the discipline j

G_k^j = expenditure of the discipline j for the k^{th} period,

F_k^j = the faculty required for the discipline j for the k^{th} period,

$B_k^{\flat}$ = the buildings of type $\flat$ required for the k^{th} period

p = the buildings for the postgraduate students,

n = the total number of disciplines taken,

N = the number of periods in the planning horizon.

The number of students called for interview is a function of many variables. These variables are listed below :

1. The number of applications received
2. Academic performance of the students in the previous examinations
3. Academic standards of the various institutions from which the applicants have graduated
4. The number of seats available for the semester for which the forecast is to be made keeping in view the number of students who would be graduating the start of this semester.

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* This occurs when the number of students who are likely to join a discipline are less than the number of seats provided for that discipline.

**This occurs when the number of students who are likely to join a discipline are more than the number of seats provided for that discipline.

5. Total hostel accommodation available. In case of completely residential institutions, the number of hostel seats vacant is a very important factor for consideration.

It is not possible to develop an explicit mathematical relationship for the exact prediction of the number of students who would be called for interview because it is a function of many intangible factors (factors 2 and 3 as listed above). The difference between the number of students to be called for interview as found by using the mathematical model and the actual number of students called for interview is called error in prediction. For all the practical purposes one can assume that this error in prediction follows a normal distribution because it is the net result of very many independent intangible factors.

The number of students who are likely to join in the various disciplines for any planning period depends upon the number of students called for interview in the past periods, the number of students joined in the past periods, salary gradient for various disciplines during the past periods and the reputation of the institution. The difference between actual and predicted value is the error in the prediction and it is assumed that it follows normal distribution.

The objective function expressed by equation (3.1) is to be optimized subject to the following constraints :

(i) Resources available to the institution

$$\sum_{j=1}^{n} G_k^j \leqslant C_k \qquad (3.2)$$

where,

C_k = total funds available to the institution in the k^{th} period.

(2) The total number of seats* sanctioned to the institution for the k^{th} period

$$\sum_{j=1}^{n} U_k^j = I_k \qquad (3.3)$$

where,

I_k = total number of seats sanctioned to the institution for the k^{th} period.

(3) The number of seats in each discipline at any period k should fall between its minimum and maximum values.

$$l_k^j \leqslant U_k^j \leqslant u_k^j \qquad (3.4)$$

where,

l_k^j and u_k^j are the minimum and maximum value on the number of seats in the discipline j for the k^{th} period.

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* The total number of seats sanctioned to the institution for any period k is a function of the total funds for giving scholarship to the students and hostel facilities.

(4) The minimum and maximum number of permissible seats in each discipline for four consecutive periods is bounded.

$$E^j_{min} \leq U^j_k + U^j_{k+1} + U^j_{k+2} + U^j_{k+3} \leq E^j_{max} \qquad (3.5)$$

where,

E^j_{min} and E^j_{max} are the minimum and maximum number of permissible seats in discipline j for four consecutive periods.

(5) Additional constraints :

(a) Constraint due to buildings - Buildings of type i required in period k (B^i_k) are less than or equal to initial stock plus new construction since beginning of planning period k.

(b) Constraint due to faculty strength - Teachers with speciality in discipline j required to accommodate the enrolments in period k (F^j_k) for discipline 'j' are less than or equal to teachers surviving from the initial strength plus new teachers recruited during the planning period.

If all these restrictions are simultaneously taken into account, the problem becomes too complex to handle. In order to simplify the problem it is necessary to make a few assumptions. In the present work only the constraints 2, 3 and 4 are considered. The entire problem has been split into two phases :

The first phase handles the forecasting of the number of students who are likely to join a discipline in a given period. In the second phase the optimum number of seats to be provided in the various disciplines is determined taking into account the various constraints.

For the first phase two types of forecasting models have been developed. In the first model called linear cyclic forecaster, X_k (the number of students who are likely to join the discipline) is taken as deterministic quantity. In the second model, X_k is treated as probabilistic in nature. This model is based on the control theory concept. The details of this model are given in following section. In the chapter of Results and Discussions the results of both the models have been compared. For the second phase dynamic programming has been used for the determination of the optimum number of seats to be provided in the various disciplines.

3.2 FORECASTING THE NUMBER OF STUDENTS LIKELY TO JOIN

3.2.1 MODEL I : Linear Cyclic Forecaster

This model has been developed to forecast the number of students who are likely to join the various disciplines. In this model the number of students who are likely to join the various disciplines is considered as deterministic. The model is developed as a time series and is capable of generating demand trends having a wide variety of slopes, growth rates, cycles and variations.

The time series used in the model for generating the forecasted demand for the number of students who are likely to join the various disciplines is expressed by the equation*

$$JD(t) = A + Bt + D(\cos(\pi t/E)) + F(\cos(\pi t/G)) + H(\cos(\pi t/U)) + DEV \quad (3.6)$$

where,

- JD(t) = generated demand of the number of students who joined the discipline for period "t",
- A = the constant which represents the demand at time period zero ("y" axis intercept)
- B = Slope of "Straight line",
- D,F,H = Coefficients which determine the amplitudes of first, second and third cyclic trends respectively,
- E,G,U = Coefficients which determine the periods of first, second and third cyclic trends respectively,
- DEV = a randomly chosen deviate from a normal population having a mean of zero and a standard deviation of SIGMA. This normal distribution is truncated at three standard deviations below and above the mean.
- N = The number of available past demand data points.

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* The above equation is applicable to each of the engineering disciplines and therefore the superscript j on the variables has not been put.

The demand is generated assuming a linear cyclic growth. The maximum number of cycles occuring in the past data for each discipline is determined by using the technique of spectral analysis [31] and it was found that for each discipline a maximum of three types of cycles can be imposed on the demand trend. The arbitrary component of the time series (DEV) is calculated using Monte Carlo Simulation Technique [25] . This keeps the usual off control points in the frequency analysis of the demand pattern at the confidence level of 99.73%. Using this model a value for each parameter (A, B, C, ... etc.) can be computed for each discipline. In the present work, it is assumed that sufficient past data for the number of students who joined in the various disciplines is available, and therefore regression analysis is used for estimation of parameters. This assumption is validated in Chapter IV.

3.2.1.1 Solution Procedure

The various steps involved are as follows :

STEP I : An iterative scheme has been employed to evaluate the values E, G and U. In this scheme one takes various values of E, G and U in the range from 1 to N/2 in steps of 1/2. This is done because the periodicity of cycles have integer values and the minimum possible number of periods in a cycle is 2. Total absolute error in the forecasting in the past N periods is used as the criterion for finding their optimal values.

STEP II : Once the periods of cycles are identified, the values of other coefficients are calculated in a manner given below :

i) Redefining the variables of time series to reduce the computational efforts, as follows :

$$
\begin{aligned}
k(t) &= ID(t) - DAVG \\
k'(t) &= JD(t) - DAVG \\
h &= t - N/2
\end{aligned}
\tag{3.7}
$$

where $ID(t)$ = actual demand for period t

DAVG = average of past demands over last N periods

ii) After substituting equation (3.7) in equation (3.6), the values of the coefficients in equation (3.6) are determined by solving the set of simultaneous equations. The set of simultaneous equations is solved by Pivotal Condensation Method [26] and the values of the elemental determinants ~~matrices~~ B1, D1, E1, F1, G1 and H1 are calculated. The solution procedure is given in Appendix A. The technique of Pivotal Condensation Method finally results in the equation :

$$k'(t)*B1 - D1 + h * E1 - F1 * \mathrm{Cos}\,\frac{\pi t}{E} + G1* \mathrm{Cos}\,\frac{\pi t}{G} - H1*\mathrm{Cos}\,\frac{\pi t}{U} = 0 \tag{3.8}$$

or $$k'(t)=(D1-E1*h+F1* \mathrm{Cos}\,\frac{\pi t}{E} - G1*\mathrm{Cos}\,\frac{\pi t}{G} + H1*\mathrm{Cos}\,\frac{\pi t}{U})/B1.$$

Since $k'(t) = JD(t) - DAVG$ (by definition)

$\bar{h} = t - N/2$ (by definition)

or $$JD(t)-DAVG = (D1-E1*h+F1 \text{ Cos } \frac{\pi t}{E} - G1 \text{ Cos } \frac{\pi t}{G} + H1 \text{ Cos } \frac{\pi t}{U})/B1$$

or $$JD(t) = \frac{D1}{B1} + DAVG - \frac{E1}{B1}\left(t - \frac{N}{2}\right) + \frac{F1}{B1} \text{ Cos } \frac{\pi t}{E} - \frac{G1}{B1} \text{ Cos } \frac{\pi t}{G} + \frac{H1}{B1} \text{ Cos } \frac{\pi t}{U} \qquad (3.9)$$

On comparing the coefficients of equations (3.6) and (3.9), one gets the values of the demand parameters as follows :

$$\begin{aligned} A &= (D1 + E1 * N/2)/B1 + DAVG \\ B &= -E1/B1 \\ D &= F1/B1 \\ F &= -G1/B1 \\ H &= H1/B1 \end{aligned} \qquad (3.10)$$

STEP III : The value of AK, the standard deviation of demand, is calculated using the equation :

$$AK, \text{ standard deviation} = \sqrt{\frac{\sum^{N}(\text{Error in forecast})^2}{N-f}}, \text{ when } N > f$$

where,

f = degrees of freedom (5 in this case)

and Error in forecast = difference between actual and forecasted demand.

Since the degree of freedom is 5, therefore, to use the above mentioned method a minimum of 6 periods' demand history must be

known. As the number of past demand data increases the accuracy in the estimation of demand parameters increases.

STEP IV : The values of E, G and U which are calculated by the iterative scheme are checked by correlation and spectral analysis [31] . These techniques are being used to calculate the approximate value of periods of cyclic trends for situations when one is faced with a large amount of past data. The iterative scheme as mentioned above becomes too cumbersome to use under such circumstances.

STEP V : During each period the forecasted demand is compared with the actual demand to evaluate the effectiveness of the forecaster. Higher the value of the error, the lower is the effectiveness of the forecaster.

Having computed all the coefficients of the time series, the forecaster can be used for planning purposes.

3.2.2 MODEL II : Kalman Filter :

This model is based on control theory concepts of Kalman Filter and takes into account the probabilistic nature of the number of students who will be called for interview in the various disciplines.

Taking number of students called for interview as the observation and the number of students who are likely to join the discipline as the state of the system (model), the Kalman-Filter

model can be expressed mathematically as*

$$\overrightarrow{X_{k+1}} = [F]\overrightarrow{X_k} + \overrightarrow{g_k}\, S_k + \overrightarrow{V_k} \tag{3.11}$$

$$Y_k = h\, X_{m,k} + W_k \tag{3.12}$$

where,

Y_k = the observation at the k^{th} period (Input to the model),

X_k = the state of the system at the k^{th} period (probabilistic quantity). It is an m component vector (output of the model),

S_k = the deterministic salary gradient for the k^{th} period (Input to the model),

V_k = the noise in the state of the system at the k^{th} period,

W_k = the noise in the observation at the k^{th} period,

g_k = the coefficient of the salary gradient term at the k^{th} period,

h = the coefficient relating observation and the last component of the state vector at the k^{th} period,

$X_{m,k}$ = the m^{th} component of the state vector at the k^{th} period,

F = an mxm coefficient matrix relating two consecutive states.

Each of the terms X_{k+1}, X_k, g_k and V_k in equations (3.11) and (3.12) is a m-dimensional vector. The current system state for $k+1^{th}$ period will be mostly affected by the system state for k^{th} period and as the

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*The above set of equations (3.11) and (3.12) are applicable to each of the disciplines and therefore the superscript j on the variables has not been put.

age of the data for past periods increases, its influence on the system state for current period decreases. Therefore, it has been assumed that the system state of only past periods affects the current state of the system. For the sake of simplicity considering only 2-dimensional vector case, the equations (3.11) and (3.12) can be written as :

$$\vec{X}_{k+1} = \begin{bmatrix} 0 & 1 \\ f_1 & f_2 \end{bmatrix} \begin{bmatrix} X_{1,k} \\ X_{2,k} \end{bmatrix} + \begin{bmatrix} 0 \\ g \end{bmatrix} S_k + \begin{bmatrix} V_{1,k} \\ V_{2,k} \end{bmatrix} \qquad (3.13)$$

$$Y_k = h\, X_{2,k} + W_k \qquad (3.14)$$

where,

$X_{1,k}$ = the first component of the state vector X_k at the k^{th} period,

$X_{2,k}$ = the second component of the state vector X_k at the k^{th} period,

f_1, f_2 = elements of the coefficient matrix F.

Equations (3.13) and (3.14) constitute the fundamental equations for each discipline of the educational system under consideration.

Given the observations $Y_0, Y_1, Y_2, \ldots, Y_k$ the determination of the estimate of the system state for any period K_1, $0 \leq K_1 \leq K$, the problem of predicting the system state is called "data smoothing" problem. For $K_1 = k$ and $k_1 > k$, the problems are referred as "filtering" and "forecasting" problems respectively.

Defining,

$$L_n = p\ (Y^n,\ X^n/f_1,\ f_2,\ g,\ h) \tag{3.15}$$

$$p(X^n) \triangleq p\ (X_1,\ X_2,\ \ldots\ X_n)$$

$$\triangleq p\ (X_{1,1},\ X_{2,1},\ldots,\ X_{1,n},\ X_{2,n})$$

The maximization is performed over $X_{i,k}$, $i = 1,2,\quad k=1,\ldots n,$ f_1, f_2, g and h. On simplifying equation (3.15), the following recursive relationship can be obtained :

$$L_k = p\ (Y_k/X_{2,k})\ p(X_k/X_{k-1},\ f_1,\ f_2,\ g,\ h)\ L_{k-1} \tag{3.16}$$

Therefore log function of L_k for n periods is expressed mathematically :

$$\ln L_n = -C \sum_{j=1}^{n} \frac{(Y_j - h\ X_{2,j})^2}{2\ \sigma_W^2} - \sum_{j=1}^{n} \frac{(X_{2,j} - f_1\ X_{1,j-1} - f_2\ X_{2,j-1} - gS_{j-1})^2}{2\ \sigma_{V_2}^2} - \sum_{j=1}^{n} \frac{(X_{1,j} - X_{2,j-1})^2}{2\ \sigma_{V_1}^2} \tag{3.17}$$

where,

σ_W^2 = variance of the error in the observation,

$\sigma_{V_1}^2$, $\sigma_{V_2}^2$ = Elements of the variance in the system state

C = constant term.

The objective is to find out the <u>system states</u> and parameters. For obtaining maximum likelihood estimate of the system states and values of the parameters f_1, f_2, g and h, the joint probability function of X_k and Y_k will be maximized. For this equation(3.17) is differentiated with respect to $X_{i,k}$, $i=1,2$, $k=1,\ldots,n$, f_1, f_2, g and h. Each expression is separately equated to zero. This results in a system of non-linear equations in the variables $X_{i,k}$, $i=1,2$, $k=1,\ldots,n$, f_1, f_2, g and h. This system of non linear equations can be separated into two sets of equations, that are linear in corresponding unknowns, namely, the states and the parameters.

Performing the maximization of equation (3.17) one gets,

$$\frac{\partial}{\partial X_{i,k}} (\ln L_n) = f_1 \frac{(X_{2,k+1} - f_1 X_{1,k} - f_2 X_{2,k} - gS_k)}{\sigma V_2^2} - \frac{X_{1,k} - X_{2,k-1}}{\sigma V_1^2} = 0 \quad (3.18)$$

$$k = 1,\ldots,n-1$$

$$\frac{\partial}{\partial X_{2,k}} (\ln L_n) = h \frac{Y_n - hX_{2,k}}{\sigma_W^2} - \frac{X_{2,k} - f_1 X_{1,k-1} - f_2 X_{2,k-1} - gS_{k-1}}{\sigma V_2^2} + \frac{X_{2,k+1} - f_1 X_{1,k} - f_2 X_{2,k} - gS_k}{\sigma_{V_2}^2} f_2 + \frac{X_{1,k+1} - X_{2,k}}{\sigma_{V_1}^2} = 0 \quad (3.19)$$

$$\frac{\partial}{\partial X_{1,n}} (\ln L_n) = \frac{X_{1,n} - X_{2,n-1}}{\sigma V_1^2} = 0 \quad (3.20)$$

$$\frac{\partial}{\partial X_{2,n}}(\ln L_n) = h\,\frac{Y_n - h\,X_{2,n}}{\sigma_W^2} - \frac{X_{2,n} - f_1\,X_{1,n-1} - f_2 X_{2,n-1} - gS_{n-1}}{\sigma_{V_2}^2} = 0 \quad (3.21)$$

$$\frac{\partial}{\partial f_i}(\ln L_n) = \sum_{j=1}^{n} \frac{(X_{2,j} - f_1\,X_{1,j-1} - f_2 X_{2,j-1} - gS_{j-1})}{\sigma_{V_2}^2}\,X_{i,j-1} = 0 \quad (3.22)$$

$$\frac{\partial}{\partial g}(\ln L_n) = \sum_{j=1}^{n} \frac{(X_{2,j} - f_1 - X_{1,j-1} - f_2 X_{2,j-1} - gS_{j-1})}{\sigma_{V_2}^2}\,S_{j-1} = 0 \quad (3.23)$$

$$\frac{\partial}{\partial h}(\ln L_n) = \sum_{j=1}^{n} \frac{(Y_j - h\,X_{2,j})}{\sigma_W^2}\,X_{2,j} = 0 \quad (3.24)$$

The set of equations (3.18) to (3.24) can be separated into two sets of equations that are linear in corresponding unknowns, namely, the states and parameters f_1, f_2, g and h.

The set of equations (3.18) to (3.21) can be rewritten in the matrix form as

$$B_n\,\vec{Z}_n = \vec{Q}_n \quad (3.25)$$

where

$$\vec{Z}_n' = \left[X_1', X_2', \ldots, X_n'\right]$$

$$B_n \vec{Z}_n = \vec{Q}_n$$

	$X_{1,1}$	$X_{2,1}$	$X_{1,2}$	$X_{2,2}$	$X_{1,3}$	$X_{2,3}$	$X_{1,n-2}$	$X_{2,n-2}$	$X_{1,n-1}$	$X_{2,n-1}$	$X_{1,n}$	$X_{2,n}$	Q_n
k=1	B_1	B_2	0	B_4									Q_1
	B_2	B_3	B_5	B_6									Q_2
k=2	0	B_5	B_1	B_2	0	B_4							Q_3
	B_4	B_6	B_2	B_3	B_5	B_6							Q_4
k=n-1							0	B_5	B_1	B_2	0	B_4	Q_3
							B_4	B_6	B_2	B_3	B_5	B_6	Q_{n-1}
k=n									0	B_5	B_5	0	0
									B_4	B_6	0	B_7	Q_n

where

$$
\left.
\begin{aligned}
B_1 &= -\frac{1}{\sigma_{V_1}^2} - \frac{f_1^2}{\sigma_{V_2}^2} \\
B_2 &= -\frac{f_1\, f_2}{\sigma_{V_2}^2} \\
B_3 &= -\frac{f_2^2}{\sigma_{V_2}^2} - \frac{1}{\sigma_{V_1}^2} - \frac{1}{\sigma_{V_2}^2} - \frac{h^2}{\sigma_W^2} \\
B_4 &= \frac{f_1}{\sigma_{V_2}^2} \\
B_5 &= \frac{1}{\sigma_{V_1}^2} \\
B_6 &= \frac{f_2}{\sigma_{V_2}^2} \\
B_7 &= -\frac{h^2}{\sigma_W^2} - \frac{1}{\sigma_{V_2}^2}
\end{aligned}
\right\} \qquad (3.26)
$$

$$
\begin{aligned}
C_1 &= \frac{f_1\, g}{\sigma_{V_2}^2} - \frac{X_{2,0}}{\sigma_{V_1}^2} \\
C_2 &= -\frac{hY_1}{\sigma_W^2} - \frac{f_1\, X_{1,0}}{\sigma_{V_2}^2} - \frac{f_2\, X_{2,0}}{\sigma_{V_2}^2} - \frac{gS_0}{\sigma_{V_2}^2} + g\,\frac{S_1 f_2}{\sigma_{V_2}^2}
\end{aligned}
$$

$$\left.\begin{aligned}
r_3 &= \frac{f_1 g}{\sigma_{V_2}^2} \\
r_4 &= -\frac{hY_2}{\sigma_W^2} - \frac{gS_1}{\sigma_{V_2}^2} + g\,\frac{S_2\,f_2}{\sigma_{V_2}^2} \\
r_5 &= -\frac{hY_3}{\sigma_W^2} - \frac{gS_2}{\sigma_{V_2}^2} + \frac{gS_3 f_2}{\sigma_{V_2}^2} \\
r_{n-1} &= -\frac{hY_{n-1}}{\sigma_W^2} - \frac{gS_{n-2}}{\sigma_{V_2}^2} + g\,\frac{S_{n-1}\,f_2}{\sigma_{V_2}^2} \\
r_n &= -\frac{hY_n}{\sigma_W^2} - \frac{gS_{n-1}}{\sigma_{V_2}^2}
\end{aligned}\right\} \qquad (3.27)$$

The set of equations (3.22) to (3.24) can be rewritten in matrix form as

$$D_n \vec{p} = \vec{r}_n \qquad (3.28)$$

where

$$\vec{p} = \begin{Bmatrix} f_1 \\ f_2 \\ g \\ h \end{Bmatrix}$$

$$D_n = \begin{bmatrix} \sum_{j=1}^{n} \frac{X_{1,j-1}^2}{\sigma_{V_2}^2} & \sum_{j=1}^{n} \frac{X_{1,j-1}X_{2,j-1}}{\sigma_{V_2}^2} & \sum_{j=1}^{n} \frac{S_{j-1}X_{1,j-1}}{\sigma_{V_2}^2} & 0 \\ \sum_{j=1}^{n} \frac{X_{1,j-1}X_{2,j-1}}{\sigma_{V_2}^2} & \sum_{j=1}^{n} \frac{X_{2,j-1}^2}{\sigma_{V_2}^2} & \sum_{j=1}^{n} \frac{S_{j-1}X_{2,j-1}}{\sigma_{V_2}^2} & 0 \\ \sum_{j=1}^{n} \frac{X_{1,j-1}S_{j-1}}{\sigma_{V_2}^2} & \sum_{j=1}^{n} \frac{X_{2,j-1}S_{j-1}}{\sigma_{V_2}^2} & \sum_{j=1}^{n} \frac{S_{j-1}^2}{\sigma_{V_2}^2} & 0 \\ 0 & 0 & 0 & \sum_{j=1}^{n} \frac{X_{2,j}^2}{\sigma_W^2} \end{bmatrix} \tag{3.29}$$

$$r_n = \begin{bmatrix} \sum_{j=1}^{n} \frac{X_{2,j}X_{1,j-1}}{\sigma_{V_2}^2} \\ \sum_{j=1}^{n} \frac{X_{2,j}X_{2,j-1}}{\sigma_{V_2}^2} \\ \sum_{j=1}^{n} \frac{X_{2,j}S_{j-1}}{\sigma_{V_2}^2} \\ \sum^{n} \end{bmatrix} \tag{3.30}$$

It can be noticed that the set of equations (3.25) are linear in $X_{i,k}$ $(i=1,2,\ k=1,\ldots,n)$, and can be solved to give the values of X_k provided the values of f_1, f_2, g and h appearing on the right hand side of the equation (3.25) are known. Similarly the set of equations (3.28) are linear in f_1, f_2, g and h and requires the values of $X_{i,k}$ $(i = 1,2,\ k = 1,\ldots n)$ in order to be solved for f_1, f_2, g and h. Therefore, these equations can best be solved iteratively using a suitable numerical technique.

The application of the principle of Kalman filter to the educational planning problem has resulted in values of the system states for future periods. In the absence of any constraint the values of the system state for future periods could be taken as the number of seats to be provided at that period. However, the constraints imposed on the problem may not allow to take the values of system states as the number of seats to be provided. Therefore, a suitable stage by stage optimization technique is required to yield the final results. In the present circumstances the dynamic programming seems to be a powerful optimization technique to get the optimal decisions.

The results of Linear Cyclic Forecaster and Kalman Filter technique are given in Chapter IV.

3.3 DYNAMIC PROGRAMMING MODEL

Since the prediction of the optimum number of seats to be provided in the various disciplines for each period is a stage by stage problem of prediction subjected to various constraints, **therefore** dynamic programming has been selected as a mathematical tool for taking such decisions.

In the present dissertation the optimization is carried out using the criterion of minimizing the cost function subjected to the various constraints discussed in section 3.1. The principle of optimality as suggested by Bellman [8], is invoked in order to deduce a recurrence relation characterizing that function. A solution of recurrence relation is then sought over decision variable and the optimal decision at each stage is determined and thus the optimal value of the objective function obtained. For the sake of simplicity only three disciplines - Mechanical, Electrical and Civil Engineering have been considered.

Two algorithms are generally used for solving a dynamic programming problem, namely forward and backward algorithms. In the forward algorithm one proceeds from first to last stage. In the backward algorithm one proceeds from last stage to the first stage when nothing is specified at last stage. It is always better to work with backward algorithm.

3.3.1 Formulation of the Model :

Given the minimum and maximum number of permissible seats in each discipline, dynamic programming is used to determine the sequence of decisions which optimizes the total expected cost.

Defining :

X_k^j = the number of students likely to join the discipline j for the k^{th} period,

U_k^j = the number of seats to be provided in the discipline j for the k^{th} period (decision variable)

At any period k, X_k^j is the state of the system for the discipline j and U_k^j is the number of seats to be provided for that discipline. There are two distinct possibilities,

i) the number of students likely to join the discipline j are less than the number of seats provided for the discipline - underprediction $X_k^j < U_k^j$

ii) the number of students likely to join the discipline j are more than the number of seats provided for the discipline - overprediction $X_k^j > U_k^j$

Let C_1^j and C_2^j be the penalties for under and overpredictions for the discipline j respectively. These penalties may be linear or non-linear functions of U_k^j and X_k^j. The exact nature of their dependence on U_k^j and X_k^j will depend upon how severely an

underprediction ($U_k^j < X_k^j$) or overprediction ($X_k^j > U_k^j$) is to be penalized. For simplicity these penalties are assumed to be linear functions of U_k^j and X_k^j. Then $C_1^j\,(U_k^j - X_k^j)$ and $C_2^j\,(X_k^j - U_k^j)$ will represent the total penalties for the under and overpredictions respectively. The costs are represented graphically in Fig. 3.1. In the figure L_k^j and u_k^j represent respectively the minimum and maximum number of seats in the discipline j for the k^{th} period.

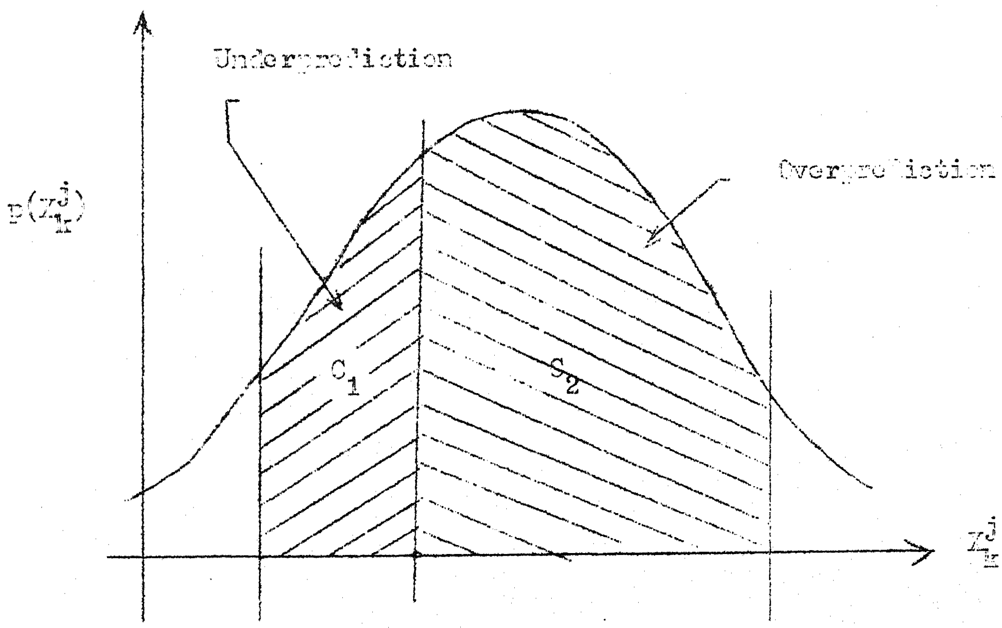

Fig. 3.1

PENALTIES FOR UNDER AND OVERPREDICTIONS

RESPECTIVELY FOR THE DISCIPLINE 'j'

At the k^{th} stage* the expected cost for underprediction in the discipline j given by $C_1^j\,(U_k^j - X_k^j)\,(U_k^j - X_k^j)\,p_k\,(X_k^j)$. The

---

* Stage and period have been used inter-changeably in this write up.

total expected cost for underprediction would be obtained by integrating between lower limit (l_k^j) and U_k^j for the discipline j. Similarly the total expected cost for overprediction for the discipline j would be obtained by integrating between U_k^j and upper limit (u_k^j). Therefore, the total expected cost of the discipline j for the k^{th} stage is given by

$$E_k^j \left[C\right] = \int_{l_k^j}^{U_k^j} C_1^j \left(U_k^j - X_k^j\right)^2 P_k(X_k^j)\, dX_k^j + \int_{U_k^j}^{u_k^j} C_2^j \left(X_k^j - U_k^j\right)^2 P_k(X_k^j)\, dX_k^j \tag{3.31}$$

$$k = 1,2,\ldots,N$$

$$j = 1,2,\ldots,n$$

where,

N = total number of stages taken

n = total number of disciplines taken

The total expected cost for discipline j over all stages $= \sum_{k=1}^{n} E_k^j \left[C\right]$ (3.32)

Applying the principle of optimality, the total optimal cost for the k^{th} stage, $E_k^*(U_k^j)$, is expressed as :

$$E_k^*(U_k^j) = \underset{U_k^j}{\mathrm{Min}} \left[C_1^j \int_{l_k^j}^{U_k^j} (U_k^j - X_k^j)^2 P_k(X_k^j) dX_k^j + C_2^j \int_{U_k^j}^{u_k^j} (X_k^j - U_k^j)^2 P_k(X_k^j) dX_k^j \right] + E_{k-1}^*(U_{k-1}^j) \tag{3.33}$$

$$j = 1,2,\ldots,n$$

and the constraints are :

i) The total number of seats to be provided in the institution at any period is bounded.

$$\sum_{j=1}^{n} U_k^j = I_k \tag{3.34}$$

where,

I_k = total number of seats sanctioned to the institution for the k^{th} period.

ii) The number of seats in each discipline at any period should fall between its minimum and maximum values.

$$l_k^j \leqslant U_k^j \leqslant u_k^j \tag{3.35}$$

where,

l_k^j and u_k^j are the lower and upper limits on the number of seats in discipline j for the k^{th} period.

iii) The minimum and maximum number of permissible seats in each discipline for four consecutive periods is bounded.

$$E_{min}^j \leqslant U_k^j + U_{k+1}^j + U_{k+2}^j + U_{k+3}^j \leqslant E_{max}^j$$

where,

E_{min}^j and E_{max}^j are the minimum and maximum number of permissible seats in discipline j for four consecutive periods.

The dynamic programming technique yields the optimum number of seats to be provided in each discipline of the institution considering the various constraints listed above. The optimization is carried out using the criterion of minimizing the cost function.

The details of the solution procedure used are given below.

3.4 Solution Procedure : The steps comprise of :

STEP I : The set of matrix equations (3.25) and (3.28) are coupled equations and cannot be solved directly. Therefore it is necessary to go in for an iterative scheme. Gauss Siedel iterative scheme with maximum pivot strategy seems to be fairly good for this purpose.

STEP II : An initial value for $\vec{p}$ is guessed. Making use of these results equation (3.25) is solved to get an estimate for $\hat{Z}_n^{(1)}$. The value of $\hat{Z}_n^{(1)}$ thus obtained is substituted in equation (3.28) and a better estimate for $\vec{p}$ is obtained. The procedure is iterated till the desired convergence is satisfied. The values of various variances are assumed to be known[27].

STEP III: The stage is upgraded and the above procedure is repeated. A better approximation for vector Z_n is obtained.

STEP IV : The scheme is iterated till the desired convergence is satisfied.

STEP V : The whole process is iterated till the two consecutive set of values of vector p obtained from two consecutive stages match to the desired accuracy.

STEP VI : Steps I to V are repeated for each discipline independently.

STEP VII: Values of f_1, f_2, g and h obtained from Step V are used directly in the set of equations (3.13) and (3.14) to forecast the number of students likely to join in future periods for each discipline individually.

STEP VIII : Keeping in view that these many students (obtained from Step VII) are now interested to join for future periods, an upper bound on the number of seats for each discipline is fixed accordingly.

STEP IX : The density function for system state for each discipline at each period is calculated using the recursive relation for variance of system state [4].

STEP X : The functional value in equation (3.33) is obtained using <u>composite trapezoidal rule</u>.

STEP XI : Finally the optimum number of seats to be provided in each discipline and the corresponding optimal cost is obtained using dynamic programming. The results of the dynamic programming are given in Chapter IV. The planning horizon of five periods is assumed.

COMPUTER PROGRAMME

Computer programmes have been developed and tested for solving all the three models described in this Chapter. A listing of the computer programme is given in Appendix C.

CHAPTER IV

RESULTS AND DISCUSSIONS

4.1 INTRODUCTION

The optimum number of seats to be provided in the various engineering disciplines have been obtained using dynamic programming. The optimization was carried out using the criterion of minimizing the cost function subjected to the various constraints as discussed in section 3.3.

4.2 DEMAND FORECASTER

4.2.1 Linear Cyclic Forecaster

The linear cyclic forecaster developed assumes a linear growth rate. Three types of seasonality factors are imposed on the single demand trend. The forecasted demand for the number of students who joined the various disciplines is calculated using the past data. Figures 4-1 through 4-3 indicate that for any academic year the number of students admitted for the first semester is always higher than the number of students admitted for the second semester.

As the decisions are based on the forecasted values and the inaccuracies in forecasting may lead, in some cases, to unprofitable policies. Results indicate that the linear cyclic growth for the forecasted demand yields very little difference between the actual and the forecasted values for the number of students who joined the various disciplines.

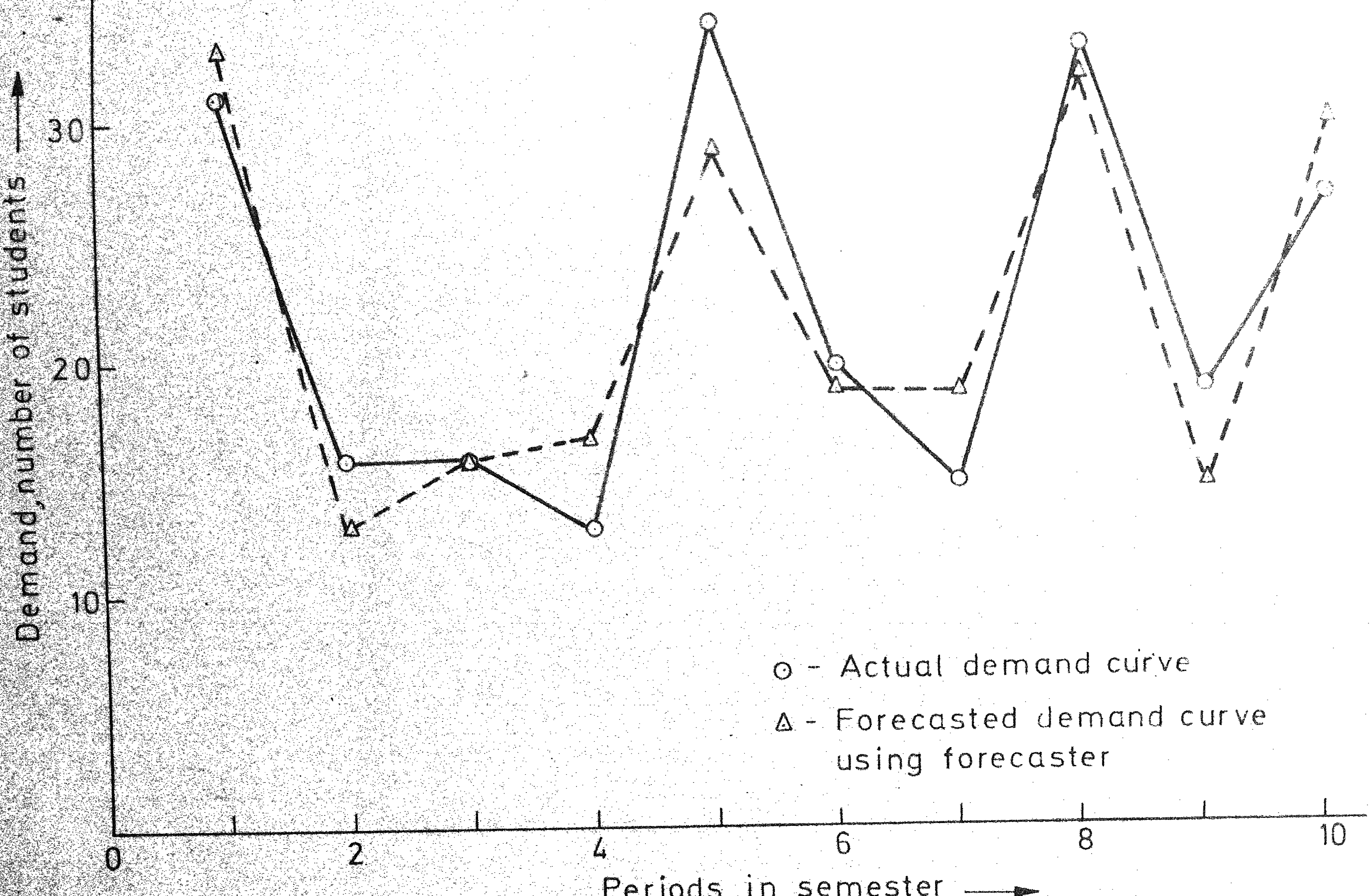

FIG. 4.1 ACTUAL AND FORECASTED DEMAND OF THE NUMBER OF STUDENTS WHO JOINED MECHANICAL ENGINEERING DISCIPLINE

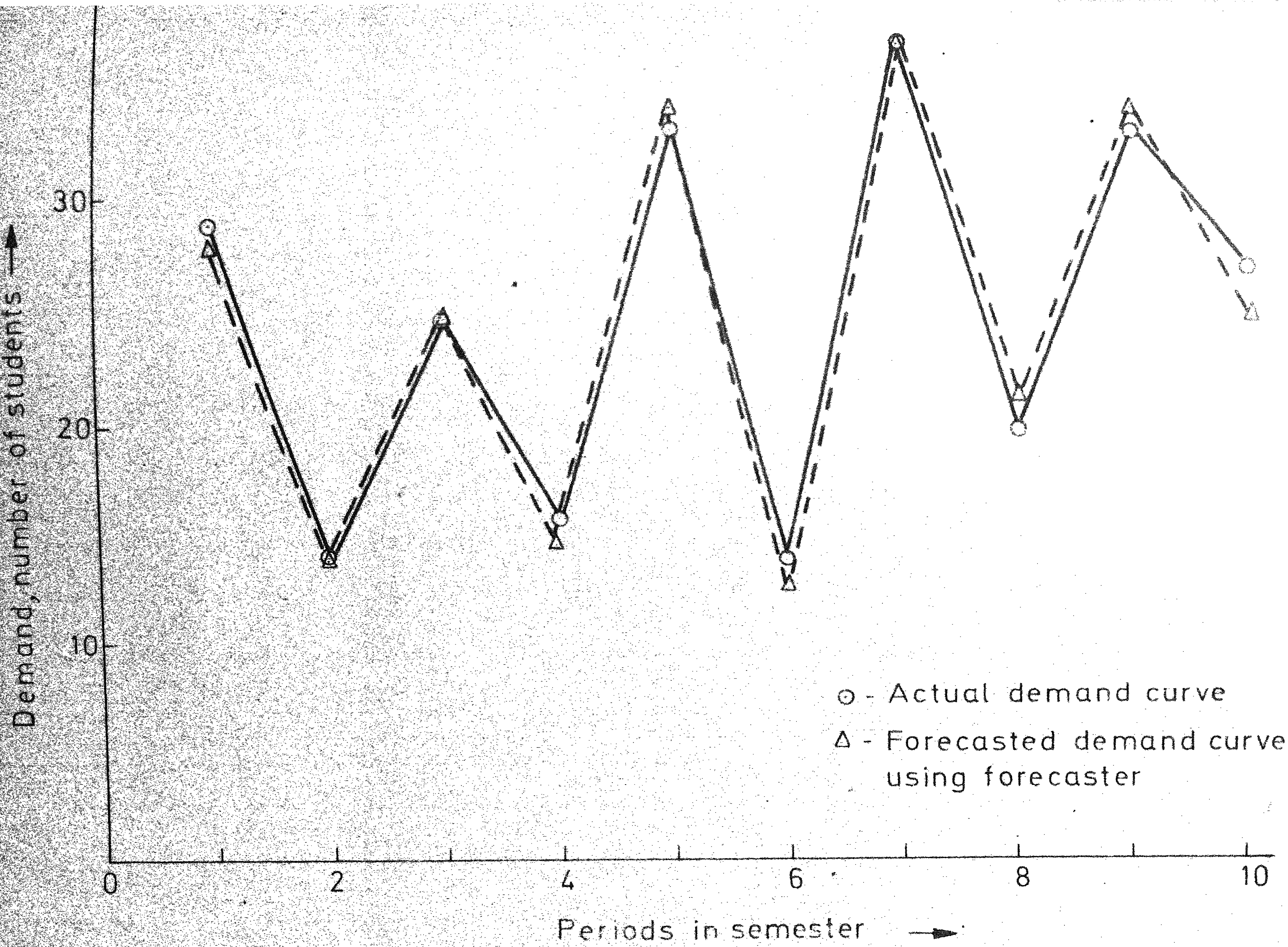

FIG.4·2 ACTUAL AND FORECASTED DEMAND OF THE NUMBER OF STUDENTS WHO JOINED ELECTRICAL ENGINEERING DISCIPLINE

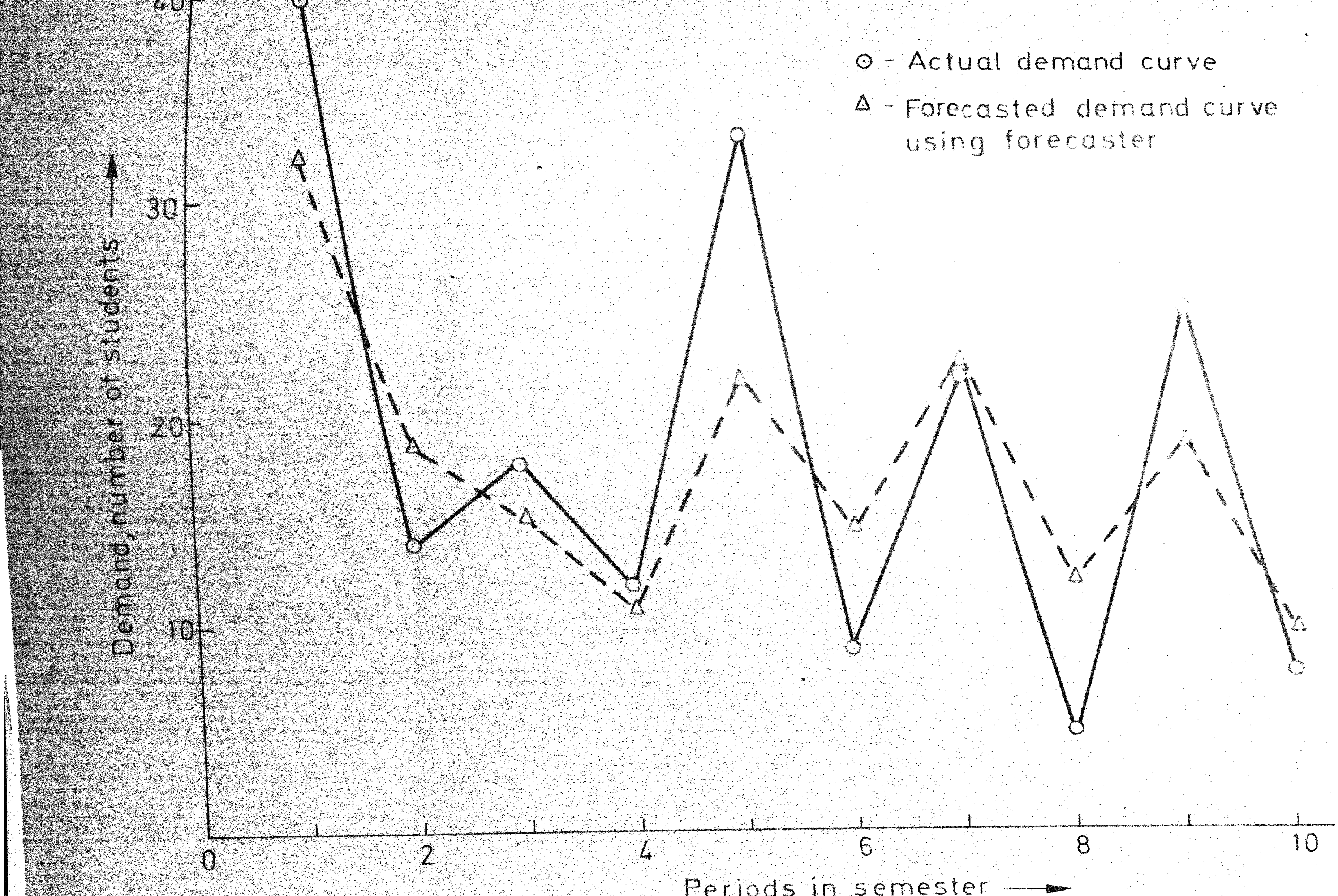

FIG. 4.3 ACTUAL AND FORECASTED DEMAND CURVE OF THE NUMBER OF STUDENTS WHO JOINED CIVIL ENGINEERING DISCIPLINE

As has already been discussed in section 3.2.1, the correlation and spectral analysis is used to check the effectiveness of the iterative procedure employed. In this technique of correlation and spectral analysis the value of lag number (m) is varied in the range of 1 to 8 by following "window closing procedure" and the subsequent values of correlation coefficient (r_k) and smooth spectral density function (G_k) are computed. The values of r_k and G_k obtained for all the ranges of m are analyzed. For significant values of m, the values are tabulated in Tables 1-A through 1-C.

Figures 4-4 through 4-6 reveal that for 95% confidence level, correlogram of past demand data does not indicate any significant periodicity. Figures 4-7 through 4-9 show that for 95% confidence level and maximum lag equal to 8, the spikes in the power spectra indicate the periodicity of 4 for mechanical engineering, periodicities of 3 and 2 for electrical engineering and periodicity of 3 for Civil Engineering. These values of periodicities are again confirmed with the iterative scheme (discussed in section 3.2.1). The forecaster parameters as obtained by the iterative scheme are tabulated in Table 2.

The results of the iterative scheme show that the total error in the forecasting for the past 10 periods is -4, 5 and 1 unit over the total actual demand values of 224, 248 and 186 for mechanical engineering, electrical engineering and civil engineering respectively.

TABLE 1-A

CORRELATION AND SPECTRAL ANALYSIS FOR MECHANICAL ENGINEERING DISCIPLINE

Maximum lag (m) = 8

S.No.	Lag k	Correlation coefficient r_k	Smooth spectral Density Function G_k	Frequency f
1	0	1.0000	4197.16	0.0000
2	1	-0.0745	1725.77	0.0625
3	2	-0.5002	14.67	0.1250
4	3	-0.3832	142.13	0.1875
5	4	0.9210	338.82	0.2500
6	5	0.0107	70.13	0.3125
7	6	-0.7023	5.00	0.3750
8	7	-0.9041	69.95	0.4375
9	8	1.0000	135.34	0.5000

TABLE 1-B

CORRELATION AND SPECTRAL ANALYSIS FOR ELECTRICAL ENGINEERING DISCIPLINE

Maximum Lag (m) = 8

S.No.	Lag k	Correlation coefficient r_k	Smooth spectral Density function G_k	Frequency f
1	0	1.0000	5223.93	0.0
2	1	-0.6697	2225.81	0.0625
3	2	0.8514	16.05	0.1250
4	3	-0.6458	6.27	0.1875
5	4	0.7841	12.30	0.2500
6	5	-0.7251	24.50	0.3125
7	6	0.9423	58.34	0.3750
8	7	-0.9530	227.55	0.4375
9	8	1.0000	100.00	0.5000

TABLE 1-C

CORRELATION AND SPECTRAL ANALYSIS FOR CIVIL ENGINEERING DISCIPLINE

Maximum Lag (m) = 8

S.No.	Lag k	Correlation coefficient r_k	Smooth spectral Density Function G_k	Frequency f
1	0	1.0000	3923.61	0.0
2	1	-0.5843	1234.93	0.0625
3	2	0.2495	5.00	0.1250
4	3	0.1131	16.38	0.1875
5	4	0.2814	79.80	0.2500
6	5	-0.1351	206.14	0.3125
7	6	-0.1617	23.25	0.3750
8	7	0.7254	337.76	0.4375
9	8	-1.0000	303.49	0.5000

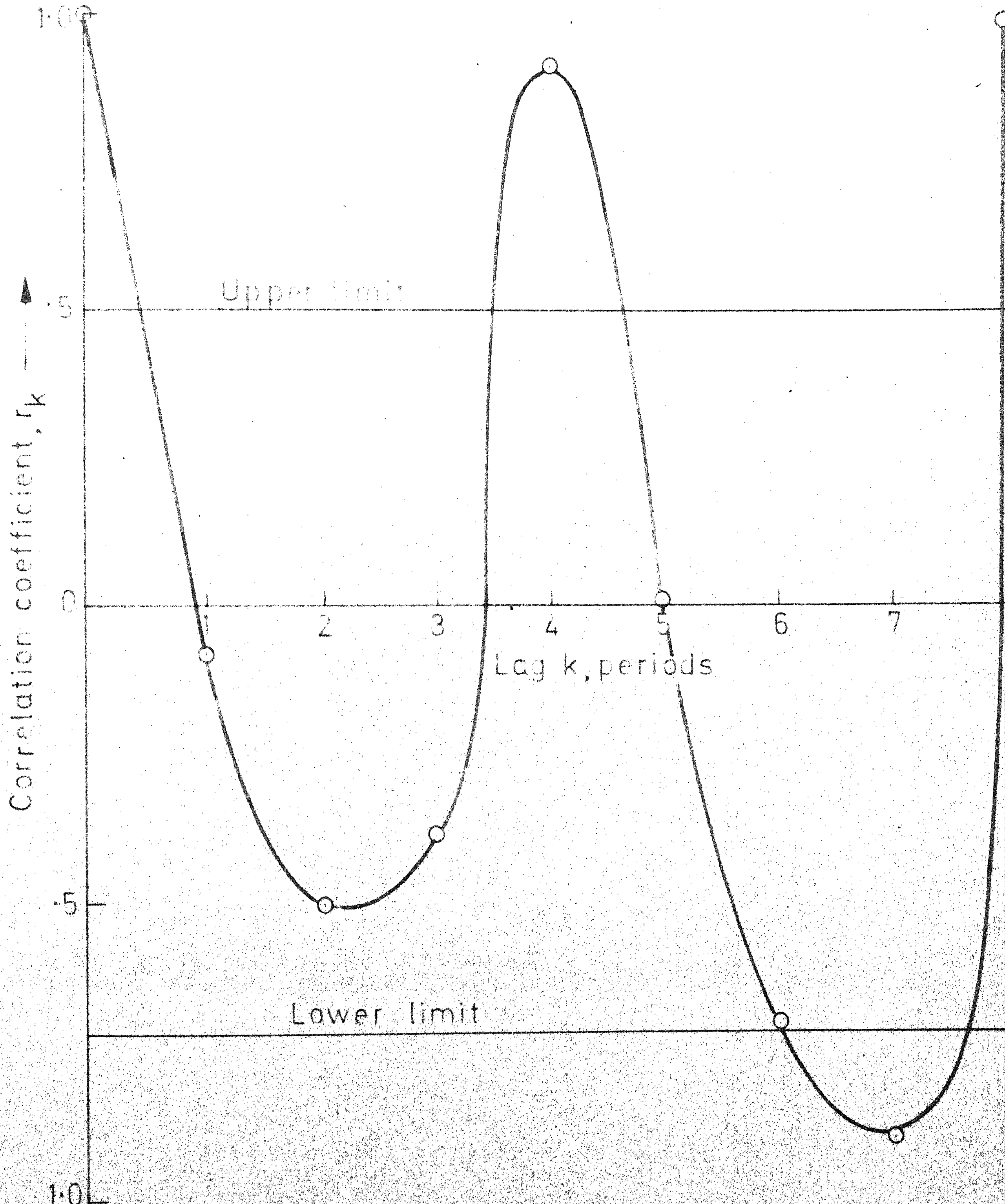

FIG. 4·4 CORRELOGRAM OF PAST DEMAND DATA OF THE NUMBER OF STUDENTS WHO JOINED MECHANICAL ENGINEERING DISCIPLINE (95 Confidence level)

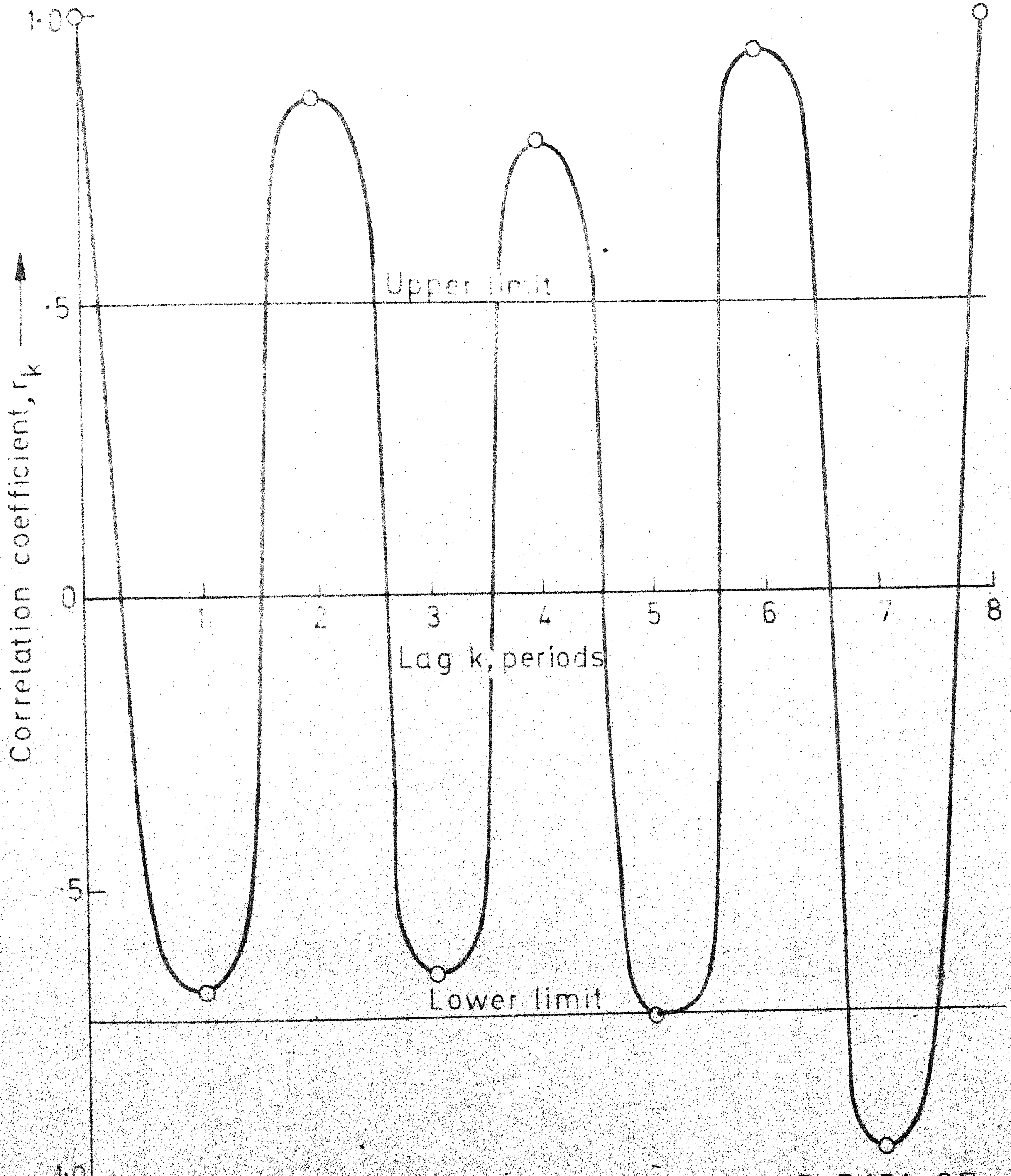

FIG. 4·5 CORRELOGRAM OF PAST DEMAND DATA OF THE NUMBER OF STUDENTS WHO JOINED ELECTRICAL ENGINEERING DISCIPLINE (95% Confidence level)

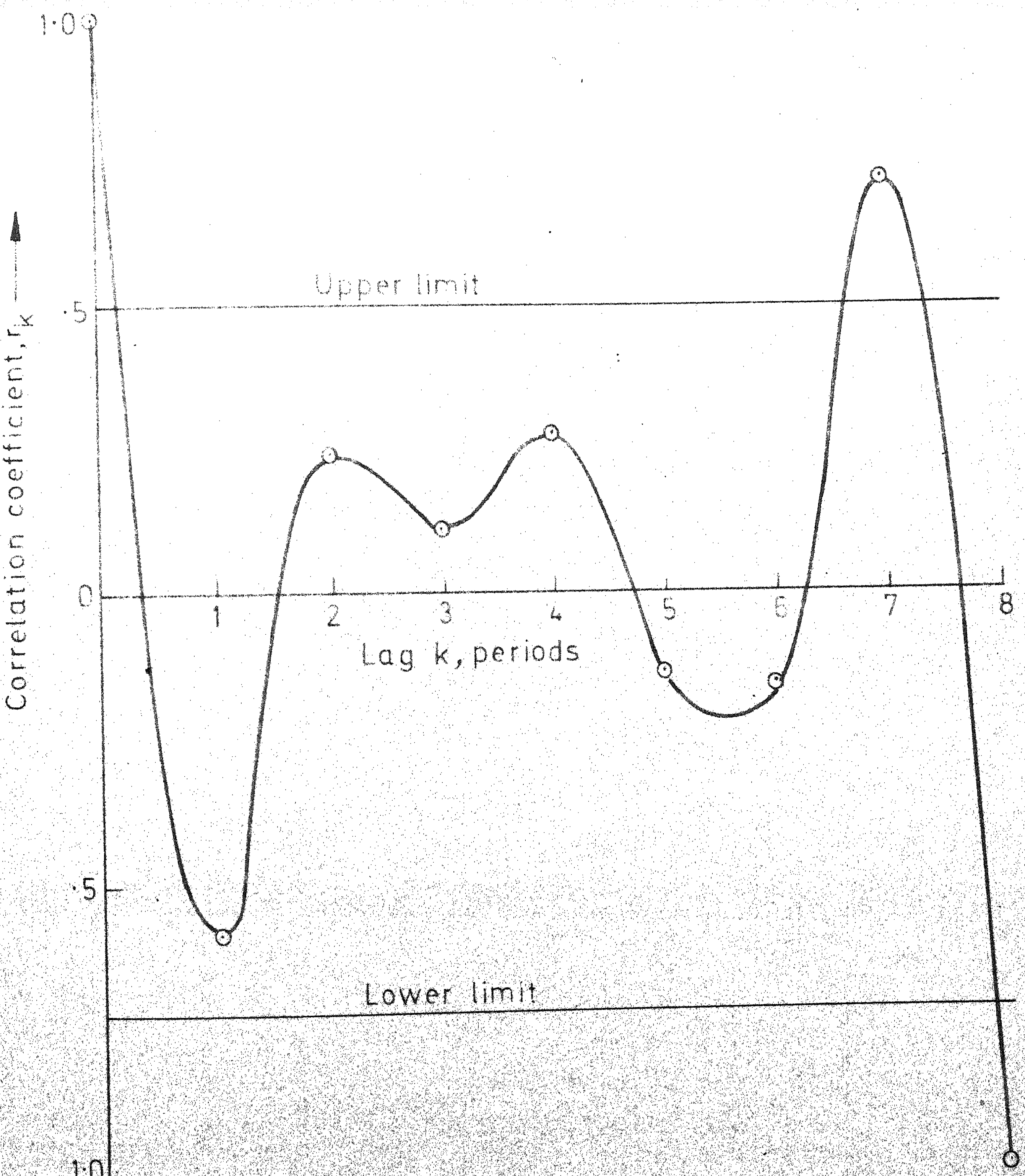

FIG. 4·6 CORRELOGRAM OF PAST DEMAND DATA OF TH
NUMBER OF STUDENTS WHO JOINED CIVIL EN
DISCIPLINE (95 % Confidence level)

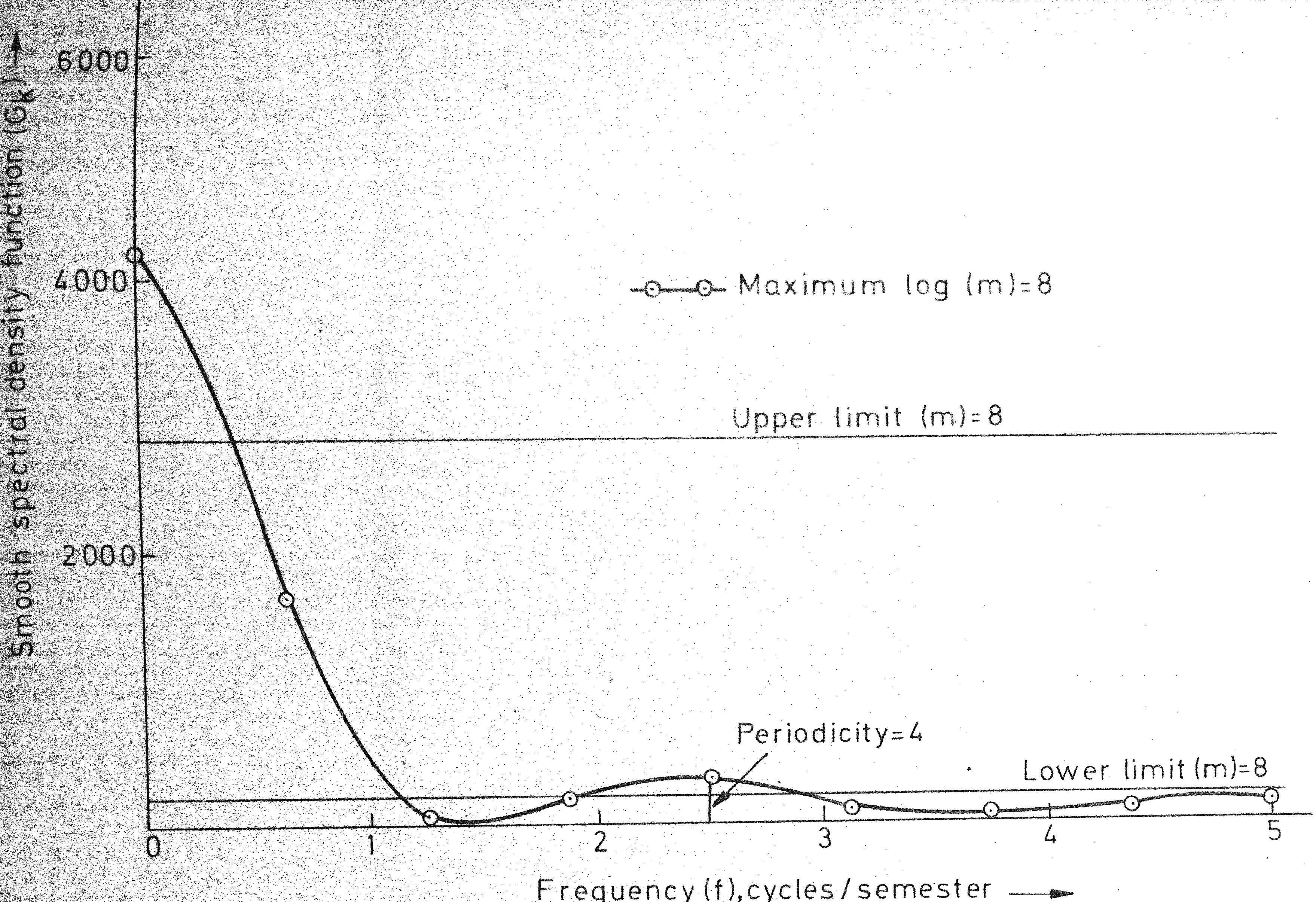

FIG. 4·7 SMOOTH SPECTRA OF PAST DEMAND DATA OF THE NUMBER OF STUDENTS WHO JOINED MECHANICAL ENGINEERING DISCIPLINE (95 % confidence level)

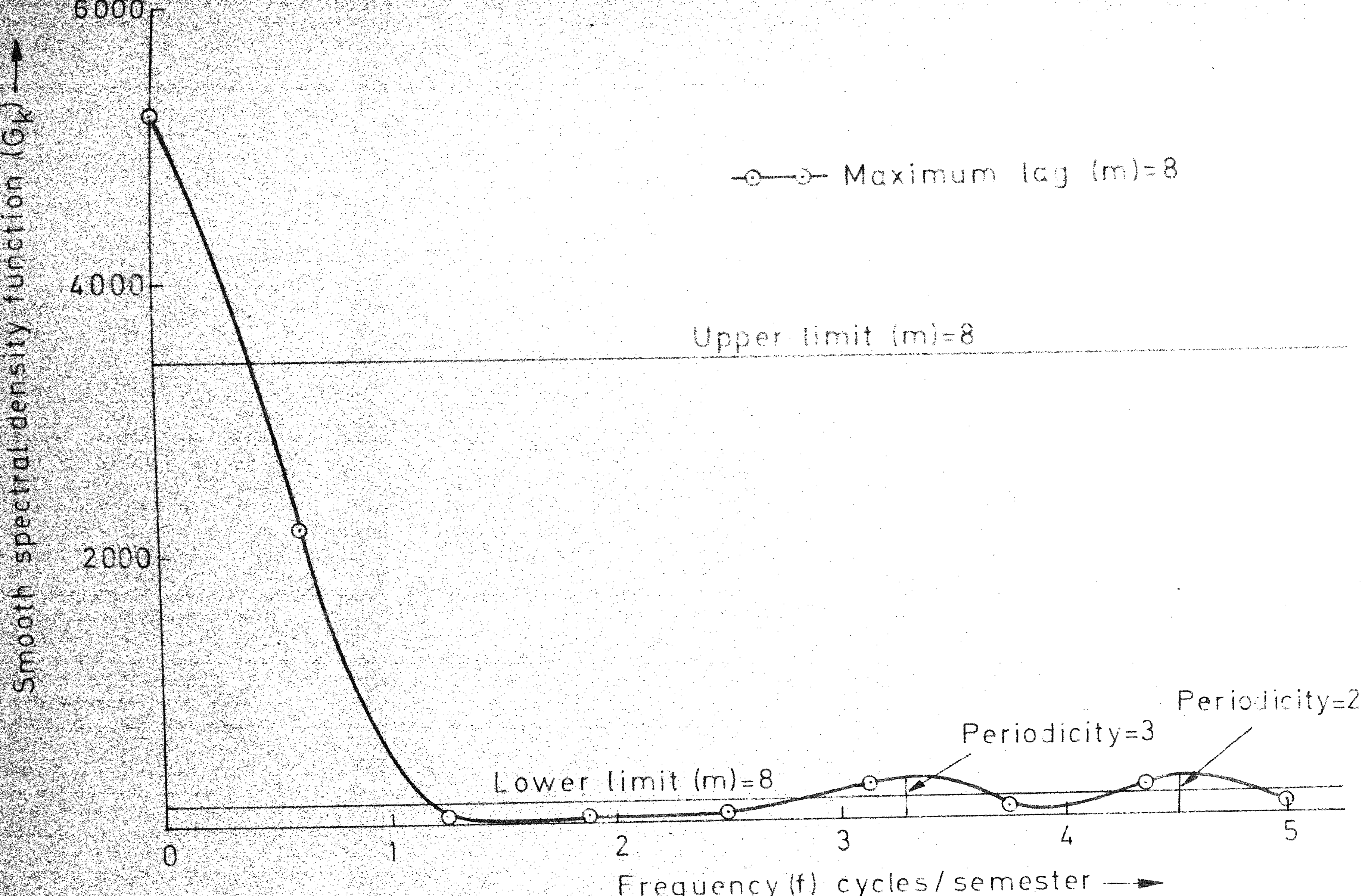

FIG. 4·8 SMOOTH POWER SPECTRA OF PAST DEMAND DATA OF THE NUMBER OF STUDENTS WHO JOINED ELECTRICAL ENGINEERING DISCIPLINE (95% Confidence level)

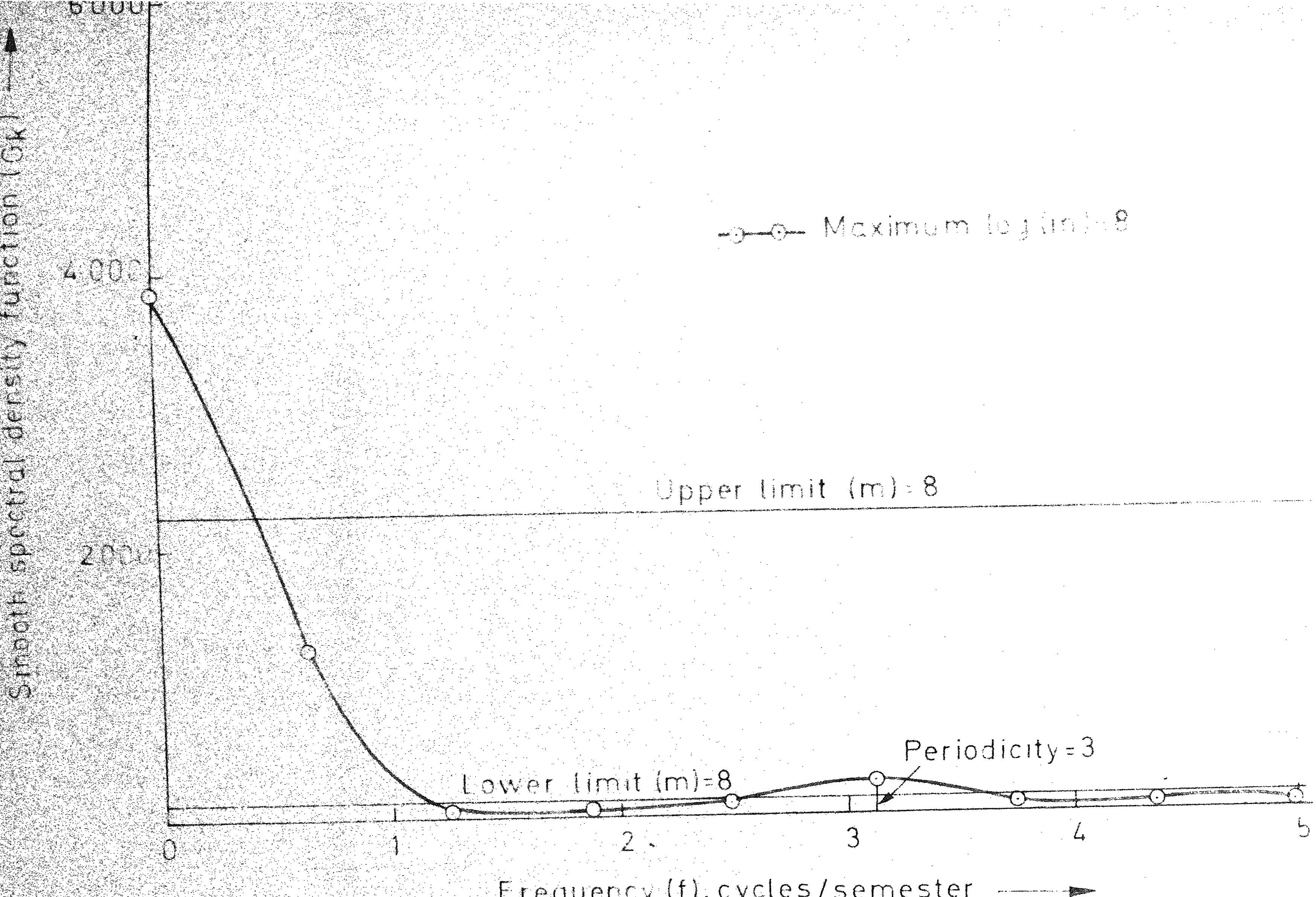

FIG 4.9 SMOOTH POWER SPECTRA OF PAST DEMAND DATA OF THE NUMBER OF STUDENTS WHO JOINED CIVIL ENGINEERING DISCIPLINE (95 % Confidence level)

TABLE 2

FORECASTER PARAMETERS

Parameters	Values for Mechanical Engineering discipline	Values for Electrical Engineering discipline	Values for Civil Engineering discipline
A	23.36	16.90	33.29
B	-0.24	1.38	- 2.76
D	5.01	-13.08	4.97
E	4.5	1.50	2.50
F	8.71	- 8.09	6.66
G	2.00	1.00	4.00
H	-3.29	9.18	5.03
J	1.00	1.50	1.50

4.2.2 Kalman Filter Forecasting Model

Three types of problems - smoothing, filtering and prediction as discussed in section 3.3 can be handled by this model. Because of its filtering property it filters the past states at each succeeding period and therefore the past states are smoothed to a great extent.

This is a probabilistic model. The principle of maximum likelihood estimate is used for predicting the optimum values of system states and parameters (discussed in Chapter III). The Gauss-Siedel-Iterative Scheme with maximum pivot strategy is employed for the evaluation of the system states and parameters respectively. Appendix B gives the flow chart elucidating the Gauss-Siedel procedure. The procedure converged in 10, 8 and 9 stages for mechanical engineering, electrical engineering and civil engineering respectively. The optimal values of Kalman-Filter parameters have been tabulated in Table 3-A and used for future forecasting.

Kalman Filter method gives a recurrence relation for the variance of the system state at each period. The total error in forecasting for the past 10 periods is -3, 2 and 1 unit for mechanical engineering, electrical engineering and civil engineering. The forecasted values of the system state for both models for planning period have been tabulated in Table 3-B.

The total error in Kalman Filter model is less than the cyclic forecaster and therefore the values obtained by Kalman Filter are adopted for planning purposes. The dynamic programming model is therefore developed on the basis of Kalman Filter.

TABLE 3-A

PARAMETERS OF KALMAN FILTER

Parameters	Values for Mechanical Engineering discipline	Values for Electrical Engineering discipline	Values for Civil Engineering discipline
f_1	0.809	0.951	0.566
f_2	-0.181	-0.235	0.035
g	0.178	0.317	0.158
h	1.746	2.418	2.513

TABLE 3-B

FORECASTED VALUES OF THE NUMBER OF STUDENTS LIKELY TO JOIN FOR FUTURE PERIODS BY LINEAR CYCLIC FORECASTER AND KALMAN-FILTER MODEL

Periods	LINEAR CYCLIC FORECASTER, GROWTH FACTOR = 0.3			KALMAN-FILTER		
	Mechanical Engineering discipline	Electrical Engineering discipline	Civil Engineering discipline	Mechanical Engg. discipline	Electrical Engineering discipline	Civil Engineering discipline
1	23	27	29	37	45	25
2	30	50	31	50	60	40
3	11	30	19	35	40	35
4	14	45	29	55	65	45
5	13	34	27	31	35	30

4.3 DYNAMIC PROGRAMMING

The optimization procedure of dynamic programming attempts to arrive at the optimum number of seats to be provided in each discipline. The output of Kalman Filter forecasting model is used as the input to dynamic programming model. Even and odd periods are specified for the first and second semester respectively. Taking the input data for problem as :

i) The maximum number of permissible seats as sanctioned by the I.I.T. Kanpur authorities for mechanical, electrical and civil engineering disciplines are 90, 110, 80 respectively.

ii) The minimum number of students needed for any discipline to exist is an administrative problem which is a function of both intangible and tangible factor. In this work the lower limit of 70, 90, 60 has been assumed for the mechanical engineering, electrical engineering and civil engineering respectively. As a matter of fact any values for the lower limit can be assumed. From the past data, it was felt that the limits stated above are quite reasonable. Further as the difference between the upper and lower limits on the permissible seats for the various disciplines increases, the computational time increases.

The optimal policy and thus the optimal cost depends upon the penalties imposed. It has been assumed that the factor γ ($\gamma = C_1/C_2$) can take any value from 0.1 onwards. To study the effect of γ on the optimal policy optimization has been carried out in the range of 0.1 to 10. Table 4 gives results for

TABLE 4

EFFECT OF CHANGE OF C_1/C_2 RATIO ON THE OPTIMAL COST

Ratio of Penalties for each Engineering discipline $\nu = C_1/C_2$	Value of Objective function in terms of units of cost (optimal cost)
0.1	0.7528
0.2	0.7034
0.4	0.6541
0.6	0.6048
0.8	0.5554
1.2	0.5061
1.8	0.4567
2.4	0.4074
4.5	0.3469
10.0	0.2312

10 different cases.

From the results, it is clear that the optimal cost decreases sharply as the value of γ increases. This is evident since increasing γ reduces the region of underprediction, as heavy penalties are imposed for underprediction. This results in U_k approaching closer and closer to X_k.

To conclude, it is observed that a value of γ equal to 10 may be an optimal one for the system under consideration. The optimal policy for the number of seats to be provided for various disciplines corresponding to γ equal to 10 has been tabulated in Table 5.

The optimal policy for the number of seats in the first semester of 1973-74 as obtained by dynamic programming for mechanical engineering, electrical engineering and civil engineering are 26, 35 and 24 respectively. The actual intake however was 20, 29 and 15 respectively. This discrepency in the results may be due to functional form of the penalties assumed. The functional form of the penalties assumed was linear while in reality it may be non-linear.

TABLE 5

RESULTS OF DYNAMIC PROGRAMMING

FOR $\lambda = 10$

Period	Optimal Policy for Mechanical Engineering discipline	Optimal Policy for Electrical Engineering discipline	Optimal Policy for Civil Engineering discipline
1	15	18	15
2	23	35	24
3	19	20	11
4	25	36	25
5	20	19	13

CHAPTER V

CONCLUSIONS AND SCOPE FOR FUTURE WORK

CONCLUSIONS

In the past, the problems pertaining to educational planning have been tackled using intution and experience. This is probably due to the fact that educational systems are very complex and difficult to handle. With the advent of high speed computer and optimization techniques. It is now possible to solve some of these problems using modern quantitative techniques. In this thesis an attempt has been made to solve the problems pertaining admissions to an educational institution using such techniques.

The proposed models have been validated using the data furnished by the Post-graduate Office of I.I.T. Kanpur. The total absolute error as found by Linear Cyclic forecaster for Mechanical Engineering, Electrical Engineering and Civil Engineering was 8.0% 3.6% and 10.0% respectively. The error obtained is small and thus the proposed forecaster is sufficiently accurate.

The author would like to point out that this research work should be considered as a beginning towards the use of modern management tools for educational planning and lot more needs to be done to bring in the necessary sophistication and reality into the proposed models.

SCOPE FOR FUTURE WORK

There are several avenues for the extension of the present work. An attempt may be made to relax some of the assumptions made in developing the model. Because of the ignorance of the exact functional form of the penalties the author assume it to be linear. However, it would be interesting to find the exact functional form of the penalties. Further the proposed model does not account for the students who leave the institution in between the terms and those who overstay. To make the model more realistic, one should study the effect of this factor on the decision policies.

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APPENDIX A

LINEAR CYCLIC FORECASTER :

In the linear cyclic forecaster the values of the coefficients A, B, D, F and H are computed using the least squares technique. In this technique a function is obtained by interpolating a set of given data. Let the function be of the form

$$JD(t) = f(t) \qquad \text{(A-1)}$$

where JD(t) is the forecasted value of the demand.

The function JD(t) should be such that it minimizes the standard error of estimate. The standard error of estimate, S_t, is defined as :

$$S_t = \sum_{1}^{N} (\text{Error in forecasting})^2 / (N-f), \text{ when } N > f \qquad \text{(A-2)}$$

where f = degrees of freedom

N = the number of available past demand data points

To minimize S_t is equivalent to minimizing E,

where $$E = \sum_{1}^{N} (\text{Error in Forecasting})^2 \qquad \text{(A-3)}$$

This amounts to minimizing the sum of the squares of the difference between the actual value of demand observed at time period t and the value that would be predicted from the forecasting function, JD(t).

The expression for E given in equation (A-3) for linear cyclic forecaster is :

$$E = \sum_{1}^{N} \left[ID(t)-A-Bt-D(\text{Cos}(\pi t/E))-F(\text{Cos}(\pi t/G))-H(\text{Cos}(\pi t/u))\right]^2 \qquad \text{(A-4)}$$

where

A = the constant which represents the demand at time period zero ("y" axis intercept),

ID(t) = actual demand for period t,

B = slope of "Straight line"

D,F,H = coefficients which determine the amplitude of first, second and third cyclic trends respectively,

E,G,U = coefficients which determine the periods of first, second and third cyclic trends respectively.

The forecaster parameters are obtained by minimizing E with respect to A, B, D, F and H. This yields,

$$\frac{\partial E}{\partial A} = \sum_{1}^{N} \left[ID(t) - \bar{A} - \bar{B}t - \bar{D}(\cos(\pi t/E)) - \bar{F}(\cos(\pi t/G)) - \bar{H}(\cos(\pi t/U)) \right] = 0 \qquad \text{(A-5)}$$

$$\frac{\partial E}{\partial B} = \sum_{1}^{N} t \left[ID(t) - \bar{A} - \bar{B}t - \bar{D}(\cos(\pi t/E)) - \bar{F}(\cos(\pi t/G)) - \bar{H}(\cos(\pi t/U)) \right] = 0 \qquad \text{(A-6)}$$

$$\frac{\partial E}{\partial D} = \sum_{1}^{N} \cos(\pi t/E) \left[ID(t) - \bar{A} - \bar{B}t - \bar{D}(\cos(\pi t/E)) - \bar{F}(\cos(\pi t/G)) - \bar{H}(\cos(\pi t/U)) \right] = 0 \qquad \text{(A-7)}$$

$$\frac{\partial E}{\partial F} = \sum_{1}^{N} \cos(\pi t/G) \left[ID(t) - \bar{A} - \bar{B}t - \bar{D}(\cos(\pi t/E)) - \bar{F}(\cos(\pi t/G)) - \bar{H}(\cos(\pi t/U)) \right] = 0 \qquad \text{(A-8)}$$

$$\frac{\partial E}{\partial H} = \sum_{1}^{N} \cos(\pi t/U) \left[ID(t) - \bar{A} - \bar{B}t - \bar{D}(\cos(\pi t/E)) - \bar{F}(\cos(\pi t/G)) - \bar{H}(\cos(\pi t/U)) \right] = 0 \qquad \text{(A-9)}$$

Redefining the variables of time series to reduce the computational efforts, as follows :

$$k(t) = HD(t) - \overline{DAVG}$$

$$k'(t) = JD(t) - \overline{DAVG} \qquad (A-10)$$

$$h = t - \bar{N}/2$$

where

$\overline{DAVG}$ = average of the past demands over last N periods.

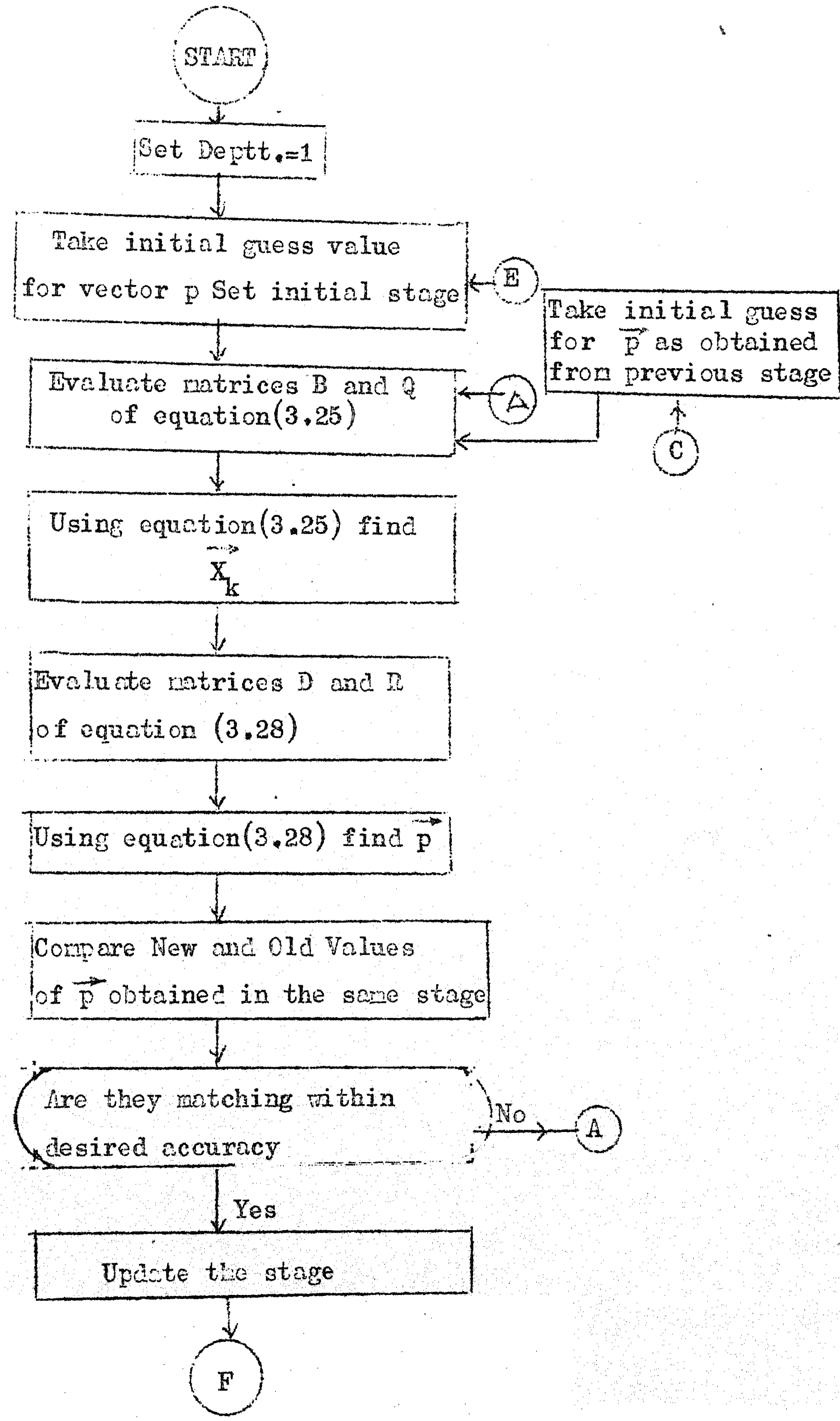
The set of simultaneous equations is solved by pivotal condensation method [26] and the values of the elemental matrices B1, D1, E1, F1, G1 and H1 are calculated. The determinant formed by these simultaneous equations is expressed as follows :

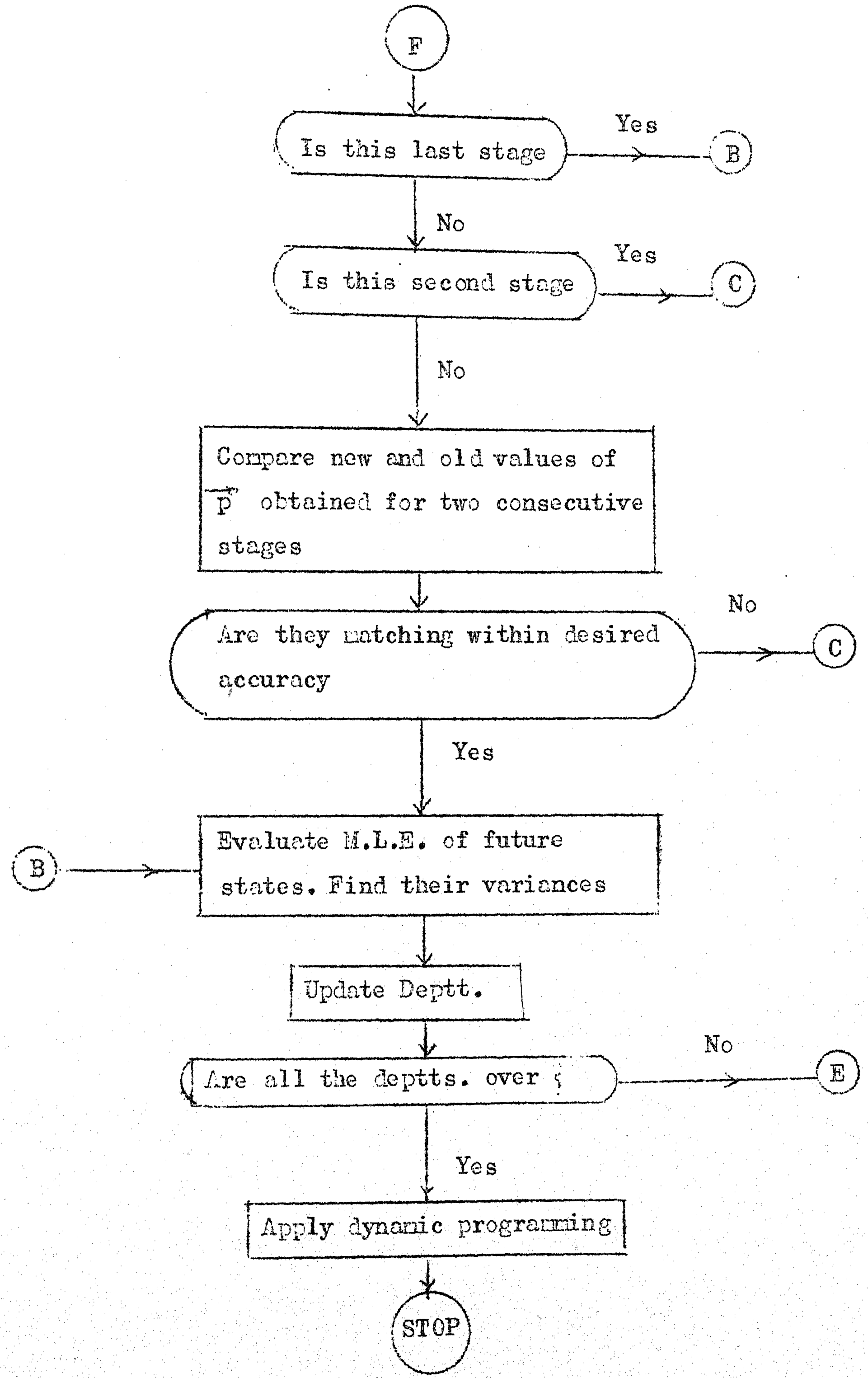
$$\begin{vmatrix}
k'(t) & 1 & h & \cos\frac{\pi t}{E} & \cos\frac{\pi t}{G} & \cos\frac{\pi t}{U} \\
\sum k(t) & N & \sum h & \sum\cos\frac{\pi t}{E} & \sum\cos\frac{\pi t}{G} & \sum\cos\frac{\pi t}{U} \\
\sum hk(t) & h & \sum h^2 & \sum h\cos\frac{\pi t}{E} & \sum h\cos\frac{\pi t}{G} & \sum h\cos\frac{\pi t}{U} \\
\sum k(t)\cos\frac{\pi t}{E} & \sum\cos\frac{\pi t}{E} & \sum h\cos\frac{\pi t}{E} & \sum\cos^2\frac{\pi t}{E} & \sum\cos\frac{\pi t}{E}\cos\frac{\pi t}{G} & \sum\cos\frac{\pi t}{E}\cos\frac{\pi t}{U} \\
\sum k(t)\cos\frac{\pi t}{G} & \sum\cos\frac{\pi t}{G} & \sum h\cos\frac{\pi t}{G} & \sum\cos\frac{\pi t}{E}\cos\frac{\pi t}{G} & \sum\cos^2\frac{\pi t}{G} & \sum\cos\frac{\pi t}{U}\cos\frac{\pi t}{G} \\
\sum k(t)\cos\frac{\pi t}{U} & \sum\cos\frac{\pi t}{U} & \sum h\cos\frac{\pi t}{U} & \sum\cos\frac{\pi t}{E}\cos\frac{\pi t}{U} & \sum\cos\frac{\pi t}{G}\cos\frac{\pi t}{U} & \sum\cos^2\frac{\pi t}{U}
\end{vmatrix}$$

The above determinant is solved by pivot condensation method [26].

APPENDIX B

FLOW CHART




F
Is this last stage
Yes
B
No
Is this second stage
Yes
C
No
Compare new and old values of $\vec{p}$ obtained for two consecutive stages
Are they matching within desired accuracy
No
C
Yes
B
Evaluate M.L.E. of future states. Find their variances
Update Deptt.
Are all the deptts. over ?
No
E
Yes
Apply dynamic programming
STOP

APPENDIX C

```
IBFTC
      ***********************************************************************

 *** THIS IS A FORECASTER PROGRAM ************************************
 *** PURPOSE - TO GENERATE FORECASTER PARAMETERS.*

      PROGRAMME FOR LINEAR,CYCLIC DEMAND FORECASTER
 *** DESCRIPTION OF INPUT PARAMETERS -
      N=DN=TOTAL NO. OF PAST PERIODS
      ID(J)=D=ACTUAL DEMAND IN PERIOD J

 *** DESCRIPTION OF VARIABLES -
         JD(I) - FORECASTER DEMAND FOR PERIOD I.
         IERR(I) - ERROR IN FORECASTING FOR PERIOD I.
         IERAB(I) - ABSOLUTE ERRORIN FORECASTING FOR PERIOD I.
         IERTOT - TOTAL ERROR IN FORECASTING.
         JDTOT - TOTAL FORECAST DEMAND.
         MINABT - MINIMUM ABSOLUTE ERROR IN FORECASTING.
         NN,MM,LL - NO. OF PERIODS IN THREE CYCLES (DIFFERENT).
         NNOPT,MMOPT,LLOPT - OPTIMUM NO. OF PERIODS IN EACH CYCLE .
         SIGMA - STANDARD DEVIATION.
      IDTOT=DTOT=TOTAL OF ACTUAL DEMANDS ID(J)

 *** ROUTINE USED *
       **FORCS**

      ***********************************************************************

      DIMENSION ID(100),JD(100),IERR(100),A(10,10),B(10,10),IERAB(100)
      READ 200,N
  200 FORMAT (I3)
      READ 100,  (ID(J),J=1,N)
  100 FORMAT(10I6)
      IF(ID(1).EQ.0)GO TO 500
      PRINT 101,N
  101 FORMAT (///////10X,*TOTAL NO. OF PAST DEMAND PERIODS = *,I8////)
      MINABT=0
      NM=5
      NM1=NM+1
 *** FOLLOWING LOOP CALCULATES THE OPTIMUM NO. OF PERIODS IN EACH CYCLE
      FOR MINIMUM TOTAL ABSOLUTE ERROR IN FORECASTING.
      DO 20 NN=2,N
      DO 20 MM=2,N
      IF(MM.EQ.NN)GO TO 20
      DO 19 LL=2,N
      IF(LL.EQ.MM)GO TO 19
 ***     CALL FORCS SUBROUTINE TO CALCULATE TOTAL ABSOLUTE ERROR IN
         FORECASTING.
      CALL FORCS(N,ID,JD,IERR,IERAB,A,IDTOT,JDTOT,IERTOT,
     1AA,BB,D,E,IABTOT,H,NN,F,G,MM,LL,U,NM,NM1)
```

```
    STORE MINIMUM VALUE OF TOTAL ERROR IN FORECASTING AND NO. OF
    PERIODS.

    IF(MINABT.EQ.0)GO TO 10
    IF(MINABT-IABTOT)20,10,10
10  MINABT=IABTCT
    NNOPT=NN
    MMOPT=MM
    LLOPT=LL
19  CONTINUE
20  CONTINUE
***     CALL FORCS SUBROUTINE TO GENERATE FORECASTING PARAMETERS FOR
        MINIMUM TOTAL ABSOLUTE ERROR IN FORECASTING.
    CALL FORCS(N,ID,JD,IERR,IERAB,A,IDTOT,JDTOT,IERTOT,AA,BB,D,E,MIN
   1ABT,H,NNOPT,F,G,MMOPT,LLOPT,U,NM,NM1)
    PRINT 112,NNOPT,MMOPT,LLOPT
112 FORMAT (////8X,80(1H*)////8X,*NO. OF PERIODS PER CYCLE (NN) = *,I8
   1////8X,*NO. OF PERIODS PER CYCLE (MM) = *,I8////8X,*NO. OF PERIODS
   2 PER CYCLE (LL) = *,I8////8X,80(1H*)/////)
    PRINT 102,AA,BB,D,E,F,G,H,U
102 FORMAT (//8X,41H****  THIS IS THE DEMAND FORECASTER  ****////5X,63
   1HD(T)=A+B*T+D*COS(PI*T/E)+F*COS(PI*T/G)+H*COS(PI*T/U)+VAR//
   2//8X,*A = *,E18.8,5X,*B = *,
   3E18.8,5X,*D=*,E18.8/8X,*E=*,E18.8,5X,*F=*,E1
   48.8,5X,*G = *,E18.8,5X,*H = *,E18.8/8X,*U= *,E18.8//)
    PRINT 103,(I,ID(I),JD(I),IERR(I),IERAB(I),I=1,N)
103 FORMAT (//8X,43H***THIS IS DEMAND FOR PRESENT 'N' PERIODS *****///
   1/8X,*PERIOD ACTUAL CALCULATED ERROR IN ABSOLUTE ERROR*/15X,*DEMAND
   2   DEMAND   FORECAST IN FORECAST*////(4X,5I8//))
    PRINT 104,IDTOT,JDTOT,IERTOT,MINABT
104 FORMAT (//8X,*TOTAL ACTUAL DEMAND = *,I8////8X,*TOTAL FORECASTED
   1DEMAND = *,I8////8X,*TOTAL ERROR IN FORECASTING = *,I8////8X,*TOTA
   2L ABSOLUTE ERROR IN FORECASTING = *,I8//)
    ******************************************************************

    THIS CALCULATES THE MINIMUM VALUE OF STANDARD DEVIATION.

    ******************************************************************


    IERTOT=IERR(1)
    IER2=IERR(1)**2
    IER2T=IER2
    DO 60 I=2,N
    IER2=IERR(I)**2
    IER2T=IER2T+IER2
    IERTOT=IERTCT+IERR(I)
    ISUM2=IERTOT**2
    N1=I-1
```

```
      PSIG=FLOAT((I*IER2T-ISUM2)/(I*N1))
      SIGMA=SQRT(PSIG)
      IF(I.LE.5) GO TO 60
      QSIG=FLOAT(IER2T/(I-NM))
      SIGMA1=SQRT(QSIG)
      SIGMA=AMIN1(SIGMA,SIGMA1)
   60 CONTINUE
      PRINT 120,SIGMA
  120 FORMAT (8X,*AK = *,E18.8)
      PUNCH 121,AA,BB,D,E,F,G,H,U,SIGMA
      GO TO 201
  121 FORMAT (8X,F16.6)
  500 CONTINUE
      STOP
      END
```

```
BFTC
      SUBROUTINE FORCS(N,ID,JD,IERR,IERAB,A,IDTOT,JDTOT,IERTOT,AA,BB,
     1D,E,IABTOT,H,NN,F,G,MM,LL,U,NM,NM1)

      ******************************************************************

***   THIS IS SUBROUTINE FORCS ***************************************
***   PURPOSE - TO DETERMINE MINIMUM ABSOLUTE ERROR IN FORECASTING.

      ******************************************************************

      DIMENSION ID(100),JD(100),IERR(100),A(10,10),B(10,10),IERAB(100)
      ENN=NN
      E=ENN/2.
      G=FLOAT(MM)/2.
      U=FLOAT(LL)/2.
      IDTOT=
      DO 1 J=1,N
    1 IDTOT=IDTOT+ID(J)
      DTOT=IDTOT
      DN=N
      DAVG=DTOT/DN
      DO 7 L=1,NM1
      DO 7 M=1,NM1
    7 A(L,M)=0.

      REDEFINE D AND T BY AK AND H TO DECREASE NO. OF CALCULATIONS
      DAVG= AVERAGE ACTUAL DEMAND IN N PERIODS
      AK=D-DAVG
      T=J=PERIOD
      H=T-N/2


      ******************************************************************

      REDEFINE D AND T BY AK AND H TO DECREASE NO. OF CALCULATIONS
      AK=D-DAVG
      T=J=PERIOD
      A(3,1)= TOTAL H*AK
      A(2,4)=A(4,2)=O=TOTAL COS
      A(2,3)= A(3,2)= TOTAL H
      A(2,2)=N
      A(2,1)= TOTAL AK
      H=T-N/2
      A(3,3)=A(3,3)=TOTAL H CUBE
      A(3,4)=A(4,3)=A(4,4)=N/I
      A(3,1)=TOTAL OF H SQARE*AK
      A(3,4)=TOTAL H SQUARE*COS=A(4,3)
      A(4,1)=TOTAL OF K*COS
      ******************************************************************
```

```
      DO 2 J=1,N
      D=ID(J)
      AK=D-DAVG
      T=J
      H=T-DN/2.
      CO=COS((22.*T)/(7.*E))
      CO2=COS((22.*T)/(7.*G))
      CO3=COS((22.*T)/(7.*U))
      A(2,1)=A(2,1)+AK
      A(2,3)=A(2,3)+H
      A(3,3)=A(3,3)+H*H
      A(3,1)=A(3,1)+H*AK
      A(4,1)=A(4,1)+H*H*AK
      A(4,4)=A(4,4)+H*H*CO
      A(2,4)=A(2,4)+CO
      A(3,4)=A(3,4)+H*CO
      A(4,4)=A(4,4)+CO*CO
      A(2,5)=A(2,5)+CO2
      A(3,5)=A(3,5)+H*CO2
      A(4,5)=A(4,5)+CO*CO2
      A(5,1)=A(5,1)+AK*CO2
      A(5,5)=A(5,5)+CO2*CO2
      A(2,6)=A(2,6)+CO3
      A(3,6)=A(3,6)+H*CO3
      A(4,6)=A(4,6)+CO*CO3
      A(5,6)=A(5,6)+CO2*CO3
      A(6,6)=A(6,6)+CO3*CO3
      A(6,1)=A(6,1)+AK*CO3
    2 A(4,1)=A(4,1)+AK*CO
      A(2,2)=DN
      DO 4 K=2,NM
      I=K
    3 I=I+1
      A(I,K)=A(K,I)
      IF(I-NM1)3,4,4
    4 CONTINUE
      ***************************************************************************

      DETERMINANT OF ORDER NM BY PIVOTAL CONDENSATION METHOD

      ***************************************************************************
      DO 35 II=1,NM1
      DO 6 J=1,NM
      DO 6 K=1,NM
      J1=J+1
      K1=K
      IF (K.GE.II)K1=K1+1
    6 B(J,K)=A(J1,K1)
      K=2
      L=1
```

```
      DETERM=1.
      M=0
    5 DO 10  I=K,NM
   11 IF(ABS(B(L,L)).LE.10.0E-25)GO TO 8
      GO TO 12
    8 M=M+1
      DO 9 KM=L,NM
      LM=L+M
      B(KM,L)=B(KM,LM)
      B(KM,LM)=B(KM,L)
    9 DETERM=-DETERM
      GO TO 11
   12 RATIO=B(I,L)/B(L,L)
      DO 10  J=K,NM
   10 B(I,J)=B(I,J)-B(L,J)*RATIO
      IF (K-NM) 15,20,20
   15 L=K
      K=K+1
      GO TO 5
   20 DO 25 L=1,NM
   25 DETERM=DETERM*B(L,L)
      GO TO (30,32,33,34,36,37),II
   30 B1=DETERM
   32 D1=DETERM
   33 E1=DETERM
   34 F1=DETERM
   36 O1=DETERM
   37 F2=DETERM
   35 CONTINUE
      F=DN/2.
      AA=(D1+E1*F)/B1+DAVG
      BB=-E1/B1
      D=+F1/B1
      F=-O1/B1
      H=+F2/B1
      JDTOT=0
      IERTOT=0
      IABTOT=0
      DO 40 I=1,N
      AI=I
      JD(I)=AA+BB*AI+D*COS((22.*AI)/(7.*E))+.5+F*COS((22.*AI)/(7
     1.*G))+H*COS((22.*AI)/(7.*U))
      IERR(I)=JD(I)-ID(I)
      IERAB(I)=IABS(IERR(I))
      IABTOT=IABTOT+IERAB(I)
      JDTOT=JDTOT+JD(I)
   40 IERTOT=IERTCT+IERR(I)
      RETURN
      END
```

$IBFTC

                                    ****

                                    FORECASTER FOR FUTURE N PERIODS

                                    ****

```
      DIMENSION JD(100)
      ITR=0
      READ 103,BINT,BINCR,BLAST
  103 FORMAT(3F5.3)
      READ 104,SEED
  104 FORMAT(F10.8)
      READ 105,N
  105 FORMAT(I5)
  10  ITR=ITR+1
      READ 101,A,D,E,F,G,H,U,SIGMA
  101 FORMAT(5F16.8)
      PRINT 102,A,D,E,F,G,H,U,SIGMA
  102 FORMAT(//8X,*DEMAND PARAMETERS(INPUT)*/8X,25(1H*)////8X,
     163H JD(T)=A+B*T+D*COS(PI*T/E)+F*COS(PI*T/G)+H*COS(PI*T/U)   +VAR
     2     ////8X,*A = *,F16.6,5X,*D = *,F16.6,5X,*E = *,F16.6,
     35X,*F = *,F16.6//8X,*G = *,F16.6,5X,*H = *,F16.6,5X,
     4*U = *,F16.6,5X,*SIGMA = *,F16.6//)
      PRINT 201,SEED
      B=BINT
  11  CONTINUE
      IF(B.GT.BLAST)GO TO 99
      N1=N+1
      ITOTD=0
      ITOTA=0
      PRINT 106,B
      DO 79 I=1,N1
      T=I-1
      MEAN=A+B*T+D*COS((22.*T)/(7.*E))+F*COS((22.*T)/(7.*G))
     1+H*COS((22.*T)/(7.*U))+0.5
      ALL1=FLOAT(MEAN)-3.*SIGMA
      AUL1=FLOAT(MEAN)+3.*SIGMA
      CALL JAYA(MEAN,ALL1,AUL1,SIGMA,SEED,DEVT)
      JD(I)=DEVT+0.5
      IDIFF1=JD(I)-MEAN
      IABS1=IABS(IDIFF1)
      ITOTD=ITOTD+IDIFF1
      ITOTA=ITOTA+IABS1
      PRINT 107,I,MEAN,JD(I),IDIFF1,IABS1,SEED
  79  CONTINUE
      PRINT 108,ITOTD,ITOTA
```

```
      B=B+BINCR
      GO TO 11
99    CONTINUE
201   FORMAT(//8X,*INITIAL SEED = *,F15.10//)
106   FORMAT(8X,*VALUE OF GRADIENT B = *,F9.5//6X,*PERIOD MEAN
     1ACTUAL  ACTUAL  ABSOLUTE  VALUE OF*/14X,*DEMAND  DEMAND
     3DIFFERENCE DIFFERENCE  SEED*)
107   FORMAT(7X,I4,3X,I6,3I10,3X,F15.10)
108   FORMAT(8X,*TOTAL DEVIATE =*,I6,8X,*TOTAL ABSOLUTE DEVIATE = *,
     9I6)
      IF(ITR.EQ.5)STOP
      GO TO 10
      END
```

```
*IBFTC
      SUBROUTINE JAYA(LM,TLMIN,TLMAX,STD,XI,DIST)
      X1=WNDY1(XI)
      CALL SNDY1(X1)
 4    RA=RNDY1(X)
      IF(RA.LE.10.0E-15)GO TO 4
      RB=RNDY1(X)
      IF(RB.LE.10.0E-15)GO TO 4
      V=(-2.0*ALOG(RA))**0.5*COS(6.283*RB)
      VAR=V*STD
      DIST=VAR+FLOAT(LM)
      XI=RB
      IF(DIST-TLMIN)6,7,8
 6    DIST=TLMIN
 7    RETURN
 8    IF(DIST-TLMAX)7,7,9
 9    DIST=TLMAX
      RETURN
      END
```

IBFTC

```
                    *********************************************
                    PROGRAM FOR CORRELATION AND SPECTRAL ANALYSIS
                    *********************************************

*** THIS IS PREMA PROGRAM ****************************************************
    PURPOSE - TC TEST CATA FOR CORRELATION AND SPECTRAL ANALYSIS.

*** DESCRIPTION OF INPUT PARAMETERS -
        X(L,I) = PAST  DEMAND  HISTORY
         L= VARIABLE NO.
         I= SAMPLE NO.
       NSAMP - NO. OF SAMPLES.
    FOR AUTOCCRRELATIN NVAR = 1.
       MLAG - MAXIMUM LAG NUMBER (BETWEEN NSAMP/20 TO NSAMP/2).

*** DESCRIPTION OF VARIABLE -
       TP(I) - SUM OF VARIABLE JX TILL PERIOD I.
       FP(I) - SUM OF VARIABLE JX TILL PERIOD I+IP.
       SP(I) - SUM OF SQUARE OF JX TILL PERIOD I.
       GP(I) - SUM OF SQUARE OF JX TILL PERIOD I+IP.
       CP(I) - SUM OF PRODUCT OF JX TILL PERIOC I AND I+IP.
       RP(I) - SERIAL CORRELATICN COEFFICIENT BETWEEN VARIABLE JX(I)
               AND JX(I+IP).
         WHERE IP=P=LAG NO. (0,1,2,...,MLAG).
       WP(I) - AUTOCOVARIANCE FLNCTION OF ORDER I.
       FLP(I) - RAW ESTIMATE OF OF A TRUE POWER SPECTRAL DENSITY
                FUNCTION OF HARMONIC NUMBER I.
       UP(I) - SMOOTH ESTIMATE CF A TRUE POWER SPECTRAL DENSITY
               FUNCTION OF HARMCNIC NUMBER I.
       AVEG - MEAN VALUE OF RP(I).
       SIGMA - STANDARD DEVIATICN OF RP(I).
       FREQ - FREQUENCY.
       CORNU - NUMBER OF DEGREES OF FREEDCM FOR SPECTRAL CALCULATION.
       GFBAR - MEAN VALUE OF SMCOTH SPECTRAL DENSITY FUNCTION.

    DIMENSION FMT(12),X(1,100),TP(100),SP(100),FP(100),GP(100),
```

```
  1CP(100),WP(100),RP(100),UP(100),FLP(100),TITLE(10),JX(1,100)
   READ 600,TITLE
   READ 1,NSAMP,NVAR
 1 FORMAT(2I5)
   IPROB=0
838 CONTINUE
   PRINT 700
   PRINT 600,TITLE
   PRINT 700
600 FORMAT(10A6)
700 FORMAT(2X,(125(1H*))/)
   IF(NVAR) 200,200,201
200 NVAR=1
201 CONTINUE
   READ4,((X(L,I),I=1,NSAMP),L=1,NVAR)
4  FORMAT(10F6.0)
   N=NSAMP+1
   AVEG=-1./(FLOAT(NSAMP)-1.)
   SIGMA=SQRT(FLOAT(NSAMP)-2.)/FLOAT(NSAMP-1)
   TWOSIG=1.96*SIGMA
   CORUP =AVEG+TWOSIG
   CORLOW=AVEG-TWOSIG
   PRINT 101,AVEG,SIGMA,TWOSIG,CORUP,CORLOW
101 FORMAT (//2X,*PARAMETERS FOR CORRELOGRAM*//5X,*MEAN =*,F10.6,*  ST
  1D. DEV. =*,F10.6,*  TWO SIGMA =*,F10.6,*  UPPER LIMIT =*,F10.6,
  2*  LOWER LIMIT =*,F10.6//)
15 CONTINUE
   READ 3,MLAG
 3 FORMAT(I5)
   IF(MLAG.LE.0) GO TO 400
   DO 300 IX=1,NVAR
   DO 300 IY=1,NVAR
   TP(1)=0.
   SP(1)=0.
   FP(1)=0.
   GP(1)=0.
   DO 16 I=1,NSAMP
   SP(1)=SP(1)+X(IX,I)**2
   TP(1)=TP(1)+X(IX,I)
   GP(1)=GP(1)+X(IY,I)**2
   FP(1)=FP(1)+X(IY,I)
16 CONTINUE
   M1=MLAG+1
   DO 20 I=2,M1
   J=I-1
   K=NSAMP-I+2
   TP(I)=TP(J)-X(IX,J)
   SP(I)=SP(I-1)-X(IX,J)**2
   FP(I)=FP(J)-X(IY,K)
   GP(I)=GP(I-1)-X(IY,K)**2
```

```
 2  CONTINUE
 23 MLAG1=MLAG+1
    DO 30 I=1,MLAG1
    NMINP=NSAMP-I+1
    CP(I)=0.
    DO 26 J=1,NMINP
    K2=J+I-1
 26 CP(I)=CP(I)+(X(IX,K2)*X(IY,J))
    WP(I)=CP(I)/(FLOAT(NMINP))
    RNUM=FLOAT(NMINP)*CP(I)-FP(I)*TP(I)
    RDEN1=SQRT((FLOAT(NMINP)*GP(I))-FP(I)**2)
    RDEN2=SQRT((FLOAT(NMINP)*SP(I))-TP(I)**2)
 30 RP(I)=RNUM/(RDEN1*RDEN2)
    IF(IX-IY) 202,203,202
202 PRINT 150,IX,IY
150 FORMAT(2X,*CROSS CORRELATION X=*,I4,* Y= *,I4//5X,*P*,7X,*SUM X*,
   114X,*SUM Y*,14X,*SUM X SQ*,11X,*SUM Y SQ*,11X,*C PROD*,13X,*R COEF
   2*/)
    GO TO 204
203 PRINT 99,IX,IY
 99 FORMAT (//2X,*AUTOCORRELATION X=*,I3,*  Y=*,I3//5X,*P*,7X,*SUM X*,
   114X,*SUM Y*,14X,*SUM X SQ*,11X,*SUM Y SQ*,11X,*C PROD*,13X,*R COEF
   2*/)
204 CONTINUE
    DO 50 I=1,MLAG1
    IP=I-1
 50 PRINT 100,IP,TP(I),FP(I),SP(I),GP(I),CP(I),RP(I)
100 FORMAT(1H0,I5,5E19.8,F16.8)
    MLAG2=MLAG-1
 33 DO 39 I=1,MLAG1
    FLP(I)=0.
    DO 36 J=2,MLAG
 36 FLP(I)=FLP(I)+2.0*WP(J)*COS(3.1415927*FLOAT((I-1)*(J-1))/FLOAT(
   1MLAG))
 39 FLP(I)=FLP(I)+WP(1)+WP(MLAG1)*COS(3.14161*FLOAT(I-1))
    UP(1)=0.46*FLP(2)+.54*FLP(1)
    UP(MLAG1)=0.46*FLP(MLAG)+.54*FLP(MLAG1)
    DO 43 I=2,MLAG
 43 UP(I)=0.23*FLP(I-1)+.54*FLP(I)+.23*FLP(I+1)
    PRINT 98,IX,IY
 98 FORMAT (//2X,*SPECTRAL ANALYSIS OF*,I3,*  VS*,I3//*      P*,7X,*COV
   1AR*,14X,*RAW  SPECTRA*,5X,*SMOOTH SPECTRA*,5X,*FREQUENCY*//)
    TOTUP=0
    DO 75 I=1,MLAG1
    IP=I-1
    FREQ=FLOAT(IP)/(2.*FLOAT(MLAG))
    TOTUP=TOTUP+UP(I)
 75 PRINT 100,IP,WP(I),FLP(I),UP(I),FREQ
    CORNU=(2.*FLOAT(NSAMP)/FLOAT(MLAG))-2./3.
    GFBAR=TOTUP/FLOAT(MLAG1)
```

```
      PRINT 102,CCRNU,GFBAR
  102 FORMAT (//2X,*PARAMETERS FOR SPECTRAL ANALYSIS*//5X,*NU =*,F16.4,*
     1     MEAN =*,F16.2//)
  300 CONTINUE
      GO TO 15
  400 CONTINUE
      IPROB=IPROB+1
      IF(IPROB.LT.4)GO TO 338
      STOP
      END
```

```
IBFTC
          ************************************************
          PROGRAME FOR KALMAN FILTER AND DYNAMIC PROGRAMING
          ************************************************
          MAIN PROGRAMME
          ************************************************
    N*******NO. OF STAGES
    F(I)****PARAMETERS OF KALMAN FILTERING TECHNIQUE
    U(I)****SALARY GRADIENT
    Y(I)****OBSERVATIONS
    SIGV(I)*VARIENCE OF THE STATE
    SIGW****VARIANCE OF NOISE IN OBSERVATION
    HH******FRACTIONAL ERROR BETWEEN TWO CONSECUTIVE VALUES OF
            PARAMETERS IN A STAGE
    HK******FRACTIONAL ERROR BETWEEN TWO CONSECUTIVE VALUES OF
            PARAMETERS IN CONSECUTIVE STAGES
    F(I)****OBTAINED FOR TWO STAGES
    XMECH***MAXIMUM LIKELIHOOD ESTIMATE OF STATE OF MECH.DISCIPLINE
    XELECT***MAXIMUM LIKELIHOOD ESTIMATE OF STATE OF ELECT. DISCIPLINE
    XCIVIL**MAXIMUM LIKLIHOOD ESTIMATE OF STATE OF CIVIL DISCIPLINE
    SIGM****VARIANCE OF STATE OF MECH. DISCIPLINE
    SIGE****VARIANCE OF STATE OF ELECT. DISCIPLINE
    SIGC****VARIANCE OF STATE OF CIVIL DISCIPLINE
    C1 AND C2 PENALTIES FOR UNDER AND OVER PREDICTION
    Q(I)****HAVE THE USUAL MEANING WHEN REFERENCE TO EQN(   ) OF TEXT
    B(I,J)**HAVE THE USUAL MEANING WHEN REFERENCE TO EQN(   ) OF TEXT
    D(I,J)**HAVE THE USUAL MEANING WHEN REFERENCE TO EQN(   ) OF TEXT
    R(I)****HAVE THE USUAL MEANING WHEN REFERENCE TO EQN(   ) OF TEXT
    *********SUBROUTINE MITA EVALUATES THE MATRICES B AND Q
    ********SUBROUTINE RAJA EVALU  ATES MATRICES D AND R
    ********SUBROUTINE INVERT INVERTS A GIVEN MATRIX
    ********SUBROUTINE DYNAM USES DYNAMIC PROGRAMING TECHNIQUE
            TO CPTIMALITY PREDICT THE NO. OF SEATS TO BE PROVIDED IN
            VARIOUS DISCIPLINES
    ********SUBROUTINES FUN1,FUN2,FUN3,FN1,FN2,FN3  EVALUTE
            FUNCTIONAL VALUES
    ********SUBROUTINE AVERG CALCULATES THE FUTURE STATES
    ********SUBROUTINE COEFF EVALUTES THE VARIANCE FOR THE
            FUTURE STATES
    *****************************************************
    *****************************************************
```

IBFTC

```
      CALL FLUN(32  )
      COMMON/TAJ/U,Y,SIGV,SIGW
      COMMON/IITK/C1,C2
      COMMON/IIT/ISEAT,IMITA,NN3
      COMMON/AREA1/XMECH,SIGM,XELECT,SIGE,XCIVIL,SIGC
      DIMENSION ISEAT(2),IMITA(2,6),NN3(6)
      DIMENSION U(40),Y(40),SIGV(2),F(40),Q(40),B(40,40),
     1FF(4),D(40,4 )
      DIMENSION R(40),FFF(12,4),X(40)
      DIMENSION XMECH(10),SIGM(10),XELECT(10),SIGE(10),XCIVIL(10)
      DIMENSION SIGC(10)
      DIMENSION SIG(10),XX(10)
      M=4
      C1=0.3
      C2=1.3
      IT=0
  302 IT=IT+1
      N=10
      IF(IT.EQ.1)N=12
      GO TO (303,304,305),IT
  303 PRINT 306
      GO TO 301
  304 PRINT 307
      GO TO 301
  305 PRINT 308
  306 FORMAT(/50X,*FORECESTING OF MECH.ENGG.DEPTT.*/50X,37(1H*)/)
  307 FORMAT(/50X,*FORECESTING OF ELECT.ENGG.DEPTT.*/50X,37(1H*)/)
  308 FORMAT(/50X,*FORECESTING OF CIVIL.ENGG.DEPTT.*/50X,37(1H*)/)
  301 READ200,(U(I),I=1,N),(Y(I),I=1,N)
  200 FORMAT(8F10.5)
      READ201,SIGW,SIGV(1),SIGV(2)
  201 FORMAT(3F10.5)
      READ202,(F(I),I=1,4)
  202 FORMAT(4F10.5)
      PRINT250,(U(I),I=1,N)
      PRINT250,(Y(I),I=1,N)
      PRINT251,SIGW,SIGV(1),SIGV(2)
      PRINT252,(F(I),I=1,4)
  250 FORMAT(1X,12F10.5)
  251 FORMAT(10X,3F15.5)
  252 FORMAT(10X,4F15.5)
      N=1
      NN=N+1
      DO 1 I=1,40
      Q(I)=0.0
```

```
      DO 1 J=1,4
1     B(I,J)=0.0
      DO 2 I=1,40
      DO 2 J=1,4
      R(I)=1.
2     D(I,J)=0.0
      DO 3 I=1,M
3     FF(I)=F(1)
66    CALL MITA(NN,B,Q,F)
      MN=2*NN
      CALL INVERT(MN,B,Q,X)
      II=2*NN
      CALL RAJA (NN,D,R,X)
      IF(N.LE.2)GO TO 50
      MM=4
      CALL INVERT(MM,D,R,F)
      PRINT1000,N,(F(I),I=1,4)
1000   FORMAT(20X*STAGE*10X*VALUE OF FF*/20X,I4,10X,4(F15.8,5X))
      DO 60 I=1,M
      HH=(F(I)-FF(I))/FF(I)
      IF(ABS(HH).GT.0.01)GO TO 63
60    CONTINUE
      DO 65 I=1,M
      FF(I)=F(I)
65    FFF(N,I)=F(I)
      PRINT 100,(X(I),I=1,II)
100   FORMAT(50X,*VALUE  OF  X*/8(10X,F15.8,5(5X,F15.8)/))
      N=N+1
      NN=N+1
      IF(N.GT.10)GO TO 5
      IF(N.EQ.2)GO TO 66
      DO67 I=1,M
      HK=(FFF(N-1,I)-FFF(N-2,I))/FFF(N-1,I)
      IF(ABS(HK).GT.0.01)GO TO 68
67    CONTINUE
5     CONTINUE
      N1=N-1
      PRINT1000,N1,(FFF(N1,I),I=1,4)
      CALL AVERAG(X,XX,N1,FFF,SIG)
      GO TO(403,404,405),IT
403   DO 406 I=1,5
      SIGM(I)=SIG(I)
406   XMECH(I)=XX(I)
      GO TO 401
404   DO 407 I=1,5
      SIGE(I)=SIG(I)
407   XELECT(I)=XX(I)
      GO TO 401
405   DO 408 I=1,5
      SIGC(I)=SIG(I)
```

```
 408 XCIVIL(I)=XX(I)
 401 IF(IT.LT.3)GO TO 301
     PRINT4110,(XMECH(I),I=1,5),(SIGM(I),I=1,5)
     PRINT4108,(XELECT(I),I=1,5),(SIGE(I),I=1,5)
     PRINT4109,(XCIVIL(I),I=1,5),(SIGC(I),I=1,5)
4110 FORMAT(5X,*XMECH*,5F8.3,5X,*SIGM*,5F13.6)
4108 FORMAT(5X,*XELECT*,5F8.3,5X,*SIGE*,5F13.6)
4109 FORMAT(5X,*XCIVL*,5F8.3,5X,*SIGC*,5F13.6)
     READ254,(ISEAT(I),I=1,2),((IMITA(I,J),J=1,6),I=1,2),(NN3(I),I=1,6)
 254 FORMAT(20I4)
     DO 5555 I=1,11
     C1=C1+0.1
     C2=C2-0.1
     PRINT 5556,C1,C2
5556 FORMAT(10X,*C1=*,F15.8,10X,*C2=*,F15.8)
     CALL DYNAM
5555 CONTINUE
     STOP
  68 DO 69 I=1,M
  69 FF(I)=F(I)
     GO TO 66
  50 IF(N.EQ.2)GO TO 51
     F(3)=R(3)-D(3,1)*F(1)-D(3,2)*F(2)
     F(3)=F(3)/D(3,3)
     GO TO 52
  51 F(3)=R(3)-D(3,1)*F(1)-D(3,2)*F(2)
     F(3)=F(3)/D(3,3)
     F(4)=R(4)/D(4,4)
  52 DO 53 I=1,M
  53 FFF(N,I)=F(I)
     PRINT 1000,N,(F(I),I=1,4)
     N=N+1
     NN=N+1
     GO TO 66
     END
```

```
IBFTC
      SUBROUTINE MITA(NN,B,Q,F)
      COMMON/TAJ/U,Y,SIGV,SIGW
      DIMENSION U(40),Y(40),SIGV(2),B(40,40),Q(40),F(40)
      N=NN-1
      II=2*N
      H1=1.0/SIGV(1)**2
      H2=1.0/SIGV(2)**2
      H3=1.0/SIGW**2
      H4=F(1)**2
      H5=F(2)**2
      H6=F(1)*F(2)
      H7=F(4)**2
      DO 1 I=1,II
      DO 1 J=1,II
 1    B(I,J)=0.0
      B(1,1)=-H4*H2
      B(1,2)=-H6*H2
      B(2,1)=B(1,2)
      B(2,2)=-H5*H2-H7*H3-H1
      Q(1)=F(1)*F(3)*U(1)*H2
      Q(2)=-F(4)*Y(1)*H3+F(2)*F(3)*U(1)*H2
      B(1,3)=0.0
      B(1,4)=F(1)*H2
      B(2,3)=H1
      B(2,4)=F(2)*H2
      IF(NN.EQ.2)GO TO 10
      DO 20 K=2,N
      J=2*K-1
      JJ=J-2
      JM=J+1
      B(J,JJ)=0.0
      B(J,JJ+1)=H1
      B(J,JJ+2)=-H1-H4*H2
      B(J,JJ+3)=-H6*H2
      B(J,JJ+4)=0.0
      B(J,JJ+5)=F(1)*H2
      B(JM,JJ)=F(1)*H2
      B(JM,JJ+1)=F(2)*H2
      B(JM,JJ+2)=-H6*H2
      B(JM,JJ+3)=-(H5+1.0)*H2-H7*H3-H1
      B(JM,JJ+4)=H1
      B(JM,JJ+5)=F(2)*H2
      Q(J)=F(1)*F(3)*H2
      Q(JM)=-F(4)*Y(K)*H3-F(3)*U(K-1)*H2+F(3)*U(K)*F(2)*H2
 20   CONTINUE
 10   IJ=II-1
      IK=II-3
      B(IJ,IK)=0.
      B(IJ,IK+1)=-H1
```

```
B(IJ,IK+1)=H[illegible]
B(IJ,IK+2)=0.0
Q(IJ)= [illegible].
B(II,IK)=F(1)*H2
B(II,IK+1)=F(2)*H2
B(II,IK+2)=0.0
B(II,II)=-H7*H3-H3
Q(II)=-F(4)*Y([illegible]N)*H5-F(3)*U(N)*H2
RETURN
END
```

```
$IBFTC
      SUBROUTINE  RAJA(NN,D,R,X)
      COMMON/TAJ/U,Y,SIGV,SIGW
      DIMENSION U(40),Y(40),SIGV(2),D(40,40),R(40),X(40)
      SUMX1=0.0
      SUMX2=0.0
      SUMX12=0.0
      SUMX1U=0.0
      SUMX2U=0.0
      SUMUU=0.0
      SUMX2J=0.0
      AX2JX1=0.0
      AX2JX2=0.0
      AX2JU=0.0
      AYX=0.0
      H1=1.0/SIGV(2)**2
      H2=1.0/SIGW**2
      NN1=NN-1
      DO 10 I=1,NN1
      J=2*I-1
      K=J+1
      L=K+2
      SUMX1=SUMX1+X(J)**2
      SUMX2=SUMX2+X(K)**2
      SUMX12=SUMX12+X(J)*X(K)
      SUMX1U=SUMX1U+X(J)*U(I)
      SUMX2U=SUMX2U+X(K)*U(I)
      SUMUU=SUMUU+U(I)*U(I)
      SUMX2J=SUMX2J+X(K)*X(K)
      AX2JX1=AX2JX1+X(L)*X(J)
      AX2JX2=AX2JX2+X(L)*X(K)
      AX2JU=AX2JU+X(L)*U(I)
      AYX=AYX+X(K)*Y(I)
   10 CONTINUE
      D(1,1)=SUMX1*H1
      D(1,2)=SUMX12*H1
      D(1,3)=SUMX1U*H1
      D(1,4)=0.0
      D(2,1)=D(1,2)
      D(2,2)=SUMX2*H1
      D(2,3)=SUMX2U*H1
      D(2,4)=0.0
      D(3,1)=D(1,3)
      D(3,2)=D(2,3)
      D(3,3)=SUMUU*H1
      D(3,4)=0.0
      D(4,1)=0.0
      D(4,2)=0.0
      D(4,3)=0.0
      II=2*NN
```

```
D(4,4)=(SUMX2J+X(II)**2)*H2
R(1)=AX2JX1*H2
R(2)=AX2JX2*H2
R(3)=AX2JU*H2
R(4)=(AYX+X(II)*Y(NN))*H2
RETURN
END
```

```
*IBFTC
C      SUBROUTINE  FOR  INVERTING  THE  GIVEN  MATRIX
       SUBROUTINE INVERT(N,AA,B,X)
       DIMENSION  IR(30),IC(30)
       DIMENSION AA(40,40),B(40),X(40),A(40,41)
       NN=N+1
       DO 199 I=1,N
       A(I,NN)=B(I)
       DO 199 J=1,N
       A(I,J)=AA(I,J)
  199  CONTINUE
       K=1
  11   KJ=K-1
       DO 2 I=1,N
       DO 2 J=1,N
       IF(K.NE.1)GO TO 3
       PIVOT=ABS(A(1,1))
       I1=1
       I2=1
       GO TO 4
  3    DO 5 KK=1,KJ
       IF(I.EQ.IR(KK))GO TO 2
       IF(J.EQ.IC(KK))GO TO 2
  5    CONTINUE
       PIVOT=ABS(A(I,J))
       I1=I
       I2=J
       GO TO 4
  2    CONTINUE
  4    DO 6 I=1,N
       DO 6 J=1,N
       IF(K.EQ.1)GO TO 7
       DO 8 KK=1,KJ
       IF(I.EQ.IR(KK))GO TO 6
       IF(J.EQ.IC(KK))GO TO 6
  8    CONTINUE
  7    IF(PIVOT.GE.ABS(A(I,J)))GO TO 6
       PIVOT=ABS(A(I,J))
       I1=I
       I2=J
  6    CONTINUE
       IR(K)=I1
       IC(K)=I2
       IJ=N+1
       DO 9 J=1,IJ
       IF(J.EQ.I2)GO TO 9
       A(I1,J)=A(I1,J)/A(I1,I2)
  9    CONTINUE
       A(I1,I2)=1.0
       DO 10 I=1,N
```

```
      DO 10 J=1,IJ
      IF(I.EQ.IR(K))GO TO 10
      IF(J.EQ.IC(K))GO TO 10
      A(I,J)=A(I,J)-A(I,I2)*A(I1,J)
10    CONTINUE
      DO 89 I=1,N
      IF(I.EQ.I1)GO TO 89
      A(I,I2)=0.0
89    CONTINUE
      K=K+1
      IF(K.LE.N)GO TO 11
      DO 12 K=1,N
      IL=IR(K)
      IM=IC(K)
      X(IM)=A(IL,NN)
12    CONTINUE
      RETURN
      END
```

```
$IBFTC
C     SUBROUTINE  DYNAM
C     SUBROUTINE  DYNAM  USING  DYNAMIC  PROGRAMMING
C     ---------------------------------------------------
      COMMON/AREA1/XMECH,SIGM,XELECT,SIGE,XCIVIL,SIGC
      DIMENSION  XMECH(10),SIGM(10),XELECT(10),SIGE(10),XCIVIL(10)
      DIMENSION  SIGC(10)
      COMMON/IITK/C1,C2
      COMMON/IIT/ISEAT,IMITA,NN3
      COMMON/AREA2/U1,U2,U3,L1,L2,L3
      COMMON/AREA3/UN
      COMMON/AREA12/N
      COMMON/AREA5 /IMN
      INTEGER  U1,U2,U3,X1,X2,X3,LU1,UU2,UU3,ROUTE,PATH,RATH
      DIMENSION PATH(5,22,22),RATH(5,22,22),ROUTE(10),F1(22,22),F2(22,
     122)
      DIMENSION  ISEAT(2),IMITA(2,6),NN3(6)
      IMN=0
      PRINT251,(ISEAT(I),I=1,2)
  251 FORMAT(10X,*ISEAT*,2I7)
      PRINT252,((IMITA(I,J),J=1,6),I=1,2)
  252 FORMAT(10X,*IMITA*,12I6)
      PRINT253,(NN3(I),I=1,6)
  253 FORMAT(10X,*NN3*,6I7)
      N=1
      IJK=1
      U1=IMITA(IJK,1)
      U2=IMITA(IJK,2)
      U3=IMITA(IJK,3)
      L1=IMITA(IJK,4)
      L2=IMITA(IJK,5)
      L3=IMITA(IJK,6)
      NN1=U1-L1+1
      NN2=L1-1
      PRINT289
  289 FORMAT(/20X,*DYNAMIC  PROGRAMING*/)
      DO  15  I=1,NN1
      J=0
      X1=I+NN2
       JSEAT=ISEAT(IJK)-X1
      X2=L2
   11 X3=JSEAT-X2
      IF(X3.GT.U3)GO  TO  103
      IF(X3.LT.L3)GO  TO  10
      IF(X2.GT.U2)GO  TO  10
      JK=X2-L2+1
      J=J+1
      F1(I,JK)=FUN1(N,X1)+FUN2(N,X2)+FUN3(N,X3)
      IMN=1
  103 CONTINUE
```

```
      X2=X2+1
      GO   TO   11
[illegible] CONTINUE
      IJK=2
13    N=N+1
      U1=IMITA(IJK,1)
      U2=IMITA(IJK,2)
      U3=IMITA(IJK,3)
      L1=IMITA(IJK,4)
      L2=IMITA(IJK,5)
      L3=IMITA(IJK,6)
      KJI=1
      IF(IJK.EQ.1)KJI=2
301   NN1=U1-L1+1
      NN2=L1-1
      DO   12   I=1,NN1
      NN=N-1
      J=1
      X1=I+NN2
      JSEAT=ISEAT(IJK)-X1
      X2=L2
14    X3=JSEAT-X2
      IF(U3.LT.X3)GO TO 155
      IF(L3.GT.X3)GO TO 15
      IF(U2.LT.X2)GO TO 15
      JK=X2-L2+1
      F2(I,JK)=FUN1(N,X1)+FUN2(N,X2)+FUN3(N,X3)
      IF((NN3(1)-X1).LT.IMITA(KJI,4))GO   TO   16
      LL1=NN3(1)-X1
      GO   TO   17
16     LL1=IMITA(KJI,4)
17     IF((NN3(2)-X1).GT.IMITA(KJI,1))GO   TO   18
      UU1=NN3(2)-X1
      GO   TO   19
18    UU1=IMITA(KJI,1)
19    IF((NN3(3)-X2).LT.IMITA(KJI,5))GO   TO   20
      LL2=NN3(3)-X2
      GO   TO   21
20    LL2=IMITA(KJI,5)
21    IF((NN3(4)-X2).GT.IMITA(KJI,2))GO   TO   22
       UU2=NN3(4)-X2
      GO   TO   23
22    UU2=IMITA(KJI,2)
23    IF((NN3(5)-X3).LT.IMITA(KJI,6))GO   TO   24
      LL3=NN3(5)-X3
      GO   TO   240
24    LL3=IMITA(KJI,6)
240   IF((NN3(6)-X3).GT.IMITA(KJI,3))GO   TO   25
      UU3=NN3(6)-X3
      GO   TO   26
```

```
25    UU3=IMITA(KJI,3)
26    IJ=0
      DO  27  K1=LL1,UU1
      II=K1+1-IMITA(KJI,4)
      JJSEAT=ISEAT(KJI)-K1
      K2=LL2
 28   K3=JJSEAT-K2
      IF(K3.GT.UU3)GO  TO 31
      IF(K3.LT.LL3)GO  TO  29
      IF(K2.GT.UU2)GO  TO  29
      JK1=K2+1-IMITA(KJI,5)
      IJ=IJ+1
      IF(IJ.GT.1)GO  TO  30
      COST=F1(II,JK1)
      ROUTE(1)=K1
      ROUTE(2)=K2
      GO  TO  31
 30   IF(COST.LE.F1(II,JK1))GO TO 31
      COST=F1(II,JK1)
      ROUTE(1)=K1
      ROUTE(2)=K2
31    K2=K2+1
      GO  TO  28
29    CONTINUE
27    CONTINUE
      F2(I,JK)=F2(I,JK)+COST
      PATH(N,I,JK)=ROUTE(1)
      RATH(N,I,JK)=ROUTE(2)
155   CONTINUE
      X2=X2+1
      GO  TO  14
15    CONTINUE
12     CONTINUE
      DO  40  I=1,NN1
      X1=I+NN2
      JSEAT=ISEAT(IJK)-X1
      X2=L2
41    X3=JSEAT-X2
      IF(X3.GT.U3)GO  TO  409
      IF(X3.LT.L3)GO  TO  40
      IF(X2.GT.U2)GO  TO  40
      JK=X2-L2+1
      F1(I,JK)=F2(I,JK)
409   CONTINUE
      X2=X2+1
      GO  TO  41
40    CONTINUE
      IF(N.EQ.5)GO  TO  1011
      IF(IJK.EQ.1)GO  TO  304
      IJK=1
```

```
      GO  TO  305
304   IJK=2
305   CONTINUE
      GO  TO  13
3..   CONTINUE
      IJ=0
      DO  50  I=1,NN1
      X1=I+NN2
      JSEAT=ISEAT(IJK)-X1
      X2=L2
51    X3=JSEAT-X2
      IF(X3.GT.U3)GO  TO  53
      IF(X3.LT.L3)GO  TO  50
      IF(X2.GT.U2)GO  TO  50
      JK=X2-L2+1
      IJ=IJ+1
      IF(IJ.GT.1)GO  TO  52
      COST=F2(1,JK)
      ROUTE(1)=X1
      ROUTE(2)=X2
      GO  TO  53
 52   IF(COST.LE.F2(I,JK))GO TO 53
      COST=F2(I,JK)
      ROUTE(1)=X1
      ROUTE(2)=X2
53    X2=X2+1
      GO  TO  51
50    CONTINUE
      PRINT100,COST,ROUTE(1),ROUTE(2)
100   FORMAT(//30X,*OPT. COST=*,F15.8,5X,*ROUTE(1)=*,I3,5X,*ROUTE(2)=*,
     1I3//)
      M1=ROUTE(1)
      M2=ROUTE(2)
      I=M1-L1+NN1
      MN=M2-U1+NN1
      M3=PATH(5,I,MN)
      M4=RATH(5,I,MN)
      I=M3-L2+NN1
      MN=M4-U2+NN1
      M5=PATH(4,I,MN)
      M6=RATH(4,I,MN)
      I=M5-L1+NN1
      MN=M6-U1+NN1
      M7=PATH(3,I,MN)
      M8=RATH(3,I,MN)
      I=M7-L2+NN1
      MN=M8-U2+NN1
      M9=PATH(2,I,MN)
      M10=RATH(2,I,MN)
      PRINT101,M1,M2,M3,M4,M5,M6,M7,M8,M9,M10
```

```
101   FORMAT(50X,*FINAL  RESULTS*//10(5X,I5)///50X,*PROBLEM  IS  OVER*)
      RETURN
      END
```

```
*IBFTC
      FUNCTION  FUN1(N,UN)
C     COMPOSITE   TRAPEZOIL   RULE   IS   USED  TO
C     TO  EVALUTE  THE   FUNCTION   VALUE
C    FUN1
C
      COMMON/IITK/C1,C2
      COMMON/AREA2/U1,U2,U3,L1,L2,L3
      INTEGER U1,U2,U3,UN,UN1,UU1
      F1=FN1(L1,UN)
      IF(L1.EQ.UN)GO TO 30
      LL1=L1+1
      DO 10  I=LL1,UN
10    F1=F1+2.0*FN1(I,UN)
 30   CONTINUE
      F1=0.5*F1
      F2=FN1(U1,UN)
      IF(U1.EQ.UN)GO TO 40
      UU1=U1-1
      DO 20  I=UN,UU1
20    F2=F2+2.0*FN1(I,UN)
 40   CONTINUE
      F2=-F2/2.0
      FUN1=C1*F1+C2*F2
      RETURN
      END
```

```
*IBFTC
      FUNCTION   FUN2(M,UN)
      COMMON/IITK/C1,C2
      COMMON/AREA2/U1,U2,U3,L1,L2,L3
      INTEGER U1,L2,U3,UN,UN1,UU1,UN3,UU3,UU2,UN2
      F1=FN2(L2,UN)
      IF(L2.EQ.UN)GO TO 30
      LL2=L2+1
      DO 10 I=LL2,UN
 10   F1=F1+2.0*FN2(I,UN)
 30   CONTINUE
      F1=F1/2.
      F2=FN2(U2,UN)
      IF(U2.EQ.UN)GO TO 40
      UU2=U2-1
      DO 20 I=UN,UU2
 20   F2=F2+2.0*FN2(I,UN)
 40   CONTINUE
      F2=-F2/2.0
      FUN2=C1*F1+C2*F2
      RETURN
      END
```

```
*IBFTC
      FUNCTION  FUN3(N,UN)
      COMMON/IITK/C1,C2
      COMMON/AREA2/U1,U2,U3,L1,L2,L3
      INTEGER U1,U2,U3,UN,UN1,UU1,UN3,UU3
      F1=FN3(L3,UN)
      IF(L3.EQ.UN)GO TO 30
      LL3=L3+1
      DO 10 I=LL3,UN
 10   F1=F1+2.0*FN3(I,UN)
 30   CONTINUE
      F1=0.5*F1
      F2=FN3(U3,UN)
      IF(U3.EQ.UN)GO TO 40
      UU3=U3-1
      DO 20 I=UN,UU3
 20   F2=F2+2.0*FN3(I,UN)
 40   CONTINUE
      F2=-0.5*F2
      FUN3=C1*F1+C2*F2
      RETURN
      END
```

```
*IBFTC
      FUNCTION  FN1(X,UN)
      COMMON/AREA5 /IMN
      COMMON/AREA1/XM,SIGM,XE,SIGE,XC,SIGC
      COMMON/AREA2/U1,U2,U3,L1,L2,L3
      COMMON/AREA12/M
      DIMENSION  XMECH(10),SIGM(10),XELECT(10),SIGE(10),XCIVIL(10)
      DIMENSION  SIGC(10)
      DIMENSION  XM(10),XE(10),XC(10)
      INTEGER   UN,X,XMECH,XELECT,XCIVIL
      DO  800  I=1,5
      XMECH(I)=XM(I)
  800 CONTINUE
      N=6-M
      C1=UN-X
      C2=X-XMECH(N)
      FN1=C1*EXP(-0.5*C2**2/SIGM(N))
      RETURN
      END
```

```
*IBFTC
      FUNCTION  FN2(X,UN)
      COMMON/AREA5 /IMN
      COMMON/AREA1/XM,SIGM,XE,SIGE,XC,SIGC
      DIMENSION  XM(10),XE(10),XC(10)
      COMMON/AREA2/U1,U2,U3,L1,L2,L3
      COMMON/AREA12/M
      DIMENSION  XMECH(10),SIGM(10),XELECT(10),SIGE(10),XCIVIL(10)
      DIMENSION  SIGC(10)
      INTEGER   UN,X,XMECH,XELECT,XCIVIL
      DO  800  I=1,5
      XELECT(I)=XE(I)
 800  CONTINUE
      N=6-M
      C1=UN-X
      C2=X-XELECT(N)
      FN2=C1*EXP(-0.5*C2**2/SIGE(N))
      RETURN
       END
```

```
*IBFTC
      FUNCTION  FN3(X,UN)
      COMMON/AREA50/IMN
      COMMON/AREA1/XM,SIGM,XE,SIGE,XC,SIGC
      DIMENSION  XM(10),XE(10),XC(10)
      COMMON/AREA2/U1,U2,U3,L1,L2,L3
      COMMON/AREA12/M
      DIMENSION  XMECH(10),SIGM(10),XELECT(10),SIGE(10),XCIVIL(10)
      DIMENSION  SIGC(10)
      INTEGER   UN,X,XMECH,XELECT,XCIVIL
      DO  800  I=1,5
      XCIVIL(I)=XC(I)
800   CONTINUE
      N=6-M
      C1=UN-X
      C2=X-XCIVIL(N)
      FN3=C1*EXP(-0.5*C2**2/SIGC(N))
      RETURN
      END
```

```
*IBFTC
      SUBROUTINE   AVERAG(X,XX,N1,FFF,SIG)
      COMMON/TAJ/U,Y,SIGV,SIGW
      COMMON/AREA10/IT
      DIMENSION   X(40),XX(10),SIG(10),FFF(12,4)
      DIMENSION   UU(5),U(40),Y(40),SIGV(2)
      READ100 ,(UU(I),I=1,5)
 100  FORMAT(5F10.3)
      N=14
      IF(IT.EQ.1)N=16
      NN1=2*(N1+1)
      NN=2*N
      MN=11
      IF(IT.EQ.1)MN=13
      MM=2*MN
      MNN= MN-2
      IF(NN1.EQ.MM)GO  TO  20
      DO  10   I=N1,MNN
      IJ=2*I+2
      N2=I+1
      IK=IJ-1
      IL=IJ+1
      II=IJ+2
      X(IL)=X(IJ)
      X(II)=FFF(N1,1)*X(IK)+FFF(N1,2)*X(IJ)+FFF(N1,3)*U(N2)
 10   CONTINUE
 20   CONTINUE
      MK=MN-1
      DO  40  I=MK,N
      IJ=2*I+2
      J=I-MN+2
      IK=IJ-1
      IL=IJ+1
      II=IJ+2
      X(IL)=X(IJ)
      X(II)=FFF(N1,1)*X(IK)+FFF(N1,2)*X(IJ)+FFF(N1,3)*UU(J)
 40   CONTINUE
      DO  45  I=1,5
      J=2*(I+MN)
 45   XX(I)=X(J)
      PRINT101,(XX(I),I=1,5)
 101  FORMAT(10X,*VALUE  OF  XX*,5F20.5)
      CALL  COEFF(X,FFF,N1,SIG)
      RETURN
      END
```

```
*IBFTC
      SUBROUTINE  COEFF(X,FFF,N1,SIG)
      DIMENSION  X(40),FFF(12,4),P(2,2),A(2,2),B(2,2),C(2),K(2,2),F(2),
     1R(2,2),Q(2,2),SIG(10),AB(2,2)
      REAL  K
      DO  10  I=1,2
      DO  10  J=1,2
   10 P(I,J)=0.0
      A(1,1)=0.0
      A(1,2)=1.0
      A(2,1)=FFF(N1,1)
      A(2,2)=FFF(N1,2)
      DO  20  I=1,2
       DO  30  J=1,2
   30 B(I,J)=0.0
   20 B(I,I)=1.0
      C(1)=0.0
      C(2)=FFF(N1,4)
      D=1.0
      K(1,1)=3.0
       K(2,2)=12.0
      K(1,2)=0.0
      K(2,1)=0.0
      L=5.0
      IJ=0
   90 IJ=IJ+1
      H1=C(2)**2*P(2,2)+5.0
      H2=1.0/H1
      F(1)=C(2)*P(2,2)
      F(1)=F(1)*H2
      F(2)=FFF(N1,1)*C(2)*P(1,2)+FFF(N1,2)*C(2)*P(2,2)
      F(2)=F(2)*H2
      DO  40  I=1,2
      AB(I,1)=A(I,1)
   40 A(I,2)=A(I,2)-F(I)*C(2)
      DO  50  I=1,2
      DO  50  J=1,2
      Q(I,J)=0.0
      DO  60  KK=1,2
   60 Q(I,J)=Q(I,J)+P(I,KK)*AB(KK,J)
   50 CONTINUE
      DO 70 I=1,2
      DO 70 J=1,2
      R(I,J)=0.0
      DO  70  KK=1,2
   70 R(I,J)=R(I,J)+AB(I,KK)*Q(KK,J)
      DO  80  I=1,2
      DO  80  J=1,2
   80 P(I,J)=R(I,J)+K(I,J)+5.0*F(I)*F(J)
      PRINT 100,((P(I,J),J=1,2),I=1,2)
```

```
100   FORMAT(1X,*P(I,J)*,12F10.5)
      IF(IJ.LE.10)GO  TO  90
      IK=IJ-10
      SIG(IK)=P(2,2)
      IF(IJ.LT.15)GO  TO  90
      RETURN
      END
```

INPUT DATA

1. MECHANICAL ENGINEERING DISCIPLINE

Y_k : 195 98 105 74 134 56 120 11

148 71 119 10

S_k : 50 25 50 25 100 50 125 60 100

50 100 50

Variances: 5.0 3.0 12.0

2. ELECTRICAL ENGINEERING DISCIPLINE

Y_k : 142 77 150 88 157 80 136

70 120 65

S_k : 120 65 100 40 60 30 65 35

60 30

Variances: 5.0 3.0 12.0

3. CIVIL ENGINEERING DISCIPLINE

Y_k : 117 77 78 68 171 37 60 30

78 38

S_k : 75 40 60 30 50 25 60 30

55 25

4. E_{max} for Mechanical, Electrical and Civil Engineering disciplines : 90, 110, 80 respectively.

5. E_{min} For Mechanical, Electrical and Civil Engineering disciplines : 70, 90, 60 respectively.